1. Does *17* divide *357*?

Answer: 357 ÷ 17 = 21, therefore 17 | 357 (i.e. 17 divides 357)

2. Prove or disprove that if *a* | *bc*, where *a*, *b*, and *c* are positive integers, then *a*|*b* or *a*|*c*

trying to prove: if a | bc, then by definition $\exists k \in Z$ (bc = ka)
here we might notice that **bc = ka** doesn't mean that **a** is a factor of **b** or **a** is a factor of **c**,
for example **b = 4, c = 15** and **a = 6**, then **bc = 60** and **6 | 60**, but **6 | 4** and **6 | 15** - this is our *counterexample*.
Hence, we conclude that the statement is false.

3. What are the quotient and remainder when

a) *1001* is divided by *13* **b)** *-111* is divided by *11*

Solution:
a) 1001 **div** 13 = 77 and 1001 **mod** 13 = 0
b) -111 **div** 11 = -11 (1 less) and -111 **mod** 11 = -111 - 11* (-11) = -111 + 121 = 10

4. Find the values of

a) 4+5 in Z_7 **b)** $8 \cdot 5$ in Z_7 **c)** 2+3+8 in Z_9

Solution:
a) (4 + 5) mod 7 = 9 mod 7 = 2

b) (8 · 5) mod 7 = 40 mod 7 = 5
c) (2 + 3 + 8) mod 9 = 13 mod 9 = 4

5. compute (do not use a calculator)

a) $(167^{14} + 87^{13})$ **mod** 5 **b)** $(11 \cdot (-25) + 11)$ **mod** 6 **c)** 32^8 mod 6

Solution:
a) $(167^4 + 87^3)$ **mod** 5 =
= $[(167 \bmod 5)^4 + (87 \bmod 5)^3]$ **mod** 5 =
= [2^4 + 2^3] **mod** 5 =
= (16+8) **mod** 5 = 24 **mod** 5 = 4

b) $(11 \cdot (-25) + 11)$ mod 6 =
= [(11 **mod** 6) (-25 **mod** 6) + 11 **mod** 6] **mod** 6 =
= [5 · 5 + 5] **mod** 6 =
= 30 **mod** 6 = 0

c) 32^8 **mod** 6 = $(32$ **mod** $6)^8$ **mod** 6 =
= 2^8 **mod** 6 =
= [(8 mod 6)(8 mod 6)(4 mod 6)] **mod** 6 =
= [2 · 2 · 4] **mod** 6 =
=16 **mod** 6 = 4

6. A parking lot has *31* visitor spaces, numbered from *0* to *30*. visitors are assigned parking spaces using the hashing function *h(k)* = *k* ***mod*** *31*, where *k* is the number formed from the first three digits on visitor's license plate.

a) Which spaces are assigned by the hashing function to cars that have these first three digits on their license plates? 317, 918, 007, 110, 111, 310

Solution:
h(317) = 317 **mod** 31 = 7 (because 317 **div** 31 = 10, and 317 **mod** 31 = 317 - 31 * 10 = 7)
h(918) = 918 **mod** 31 = 19 (because 918 **div** 31 = 29, and 918 mod 31 = 918 - 31 * 29 = 19)
h(007) = 007 **mod** 31 = 7 - collision with car 317 (spot is already taken)
h(110) = 110 **mod** 31 = 17 (because 110 **div** 31 = 3, and 110 **mod** 31 = 110 - 31 * 3 = 17)
h(111) = 111 **mod** 31 = 18 (using the result above)
h(310) = 310 **mod** 31 = 0

b) What can you advise the visitors when the space they are assigned is occupied?

Answer:
one of the solutions is to try to occupy the next (consecutive available spot)
another - try to occupy the spot with number $\lfloor$**h(their number) / 2**$\rfloor$.

7. What sequence of pseudorandom numbers is generated using the linear congruential generator
$x_{n+1} = (5x_n+2)$ ***mod*** 9, with seed $x_0=3$?

Solution:
$x_0 = 3$
x_1 = (5*3+2) **mod** 9 = 17 **mod** 9 = 8
x_2 = (5*8+2) **mod** 9 = 42 **mod** 9 = 6
x_3 = (5*6 + 2) **mod** 9 = 32 **mod** 9 = 5
x_4 = (5*5+2) **mod** 9 = 27 **mod** 9 = 0
x_5 = (5*0 + 2) **mod** 9 = 2 **mod** 9 = 2
x_6 = (5*2 + 2) **mod** 9 = 12 **mod** 9 = 3 - from now on the pseudo-random numbers will be repeating
x_7 = (5*3+2) **mod** 9 = 17 **mod** 9 = 8 - $x_1 = x_7$
x_8 = (5*8+2) **mod** 9 = 42 **mod** 9 = 6 - $x_2 = x_8$, and so on
Generated sequence: 8, 6, 5, 0, 2, 3, 8, 6, 5, 0, 2, 3, 8, 6, 5, 0, 2, 3,...