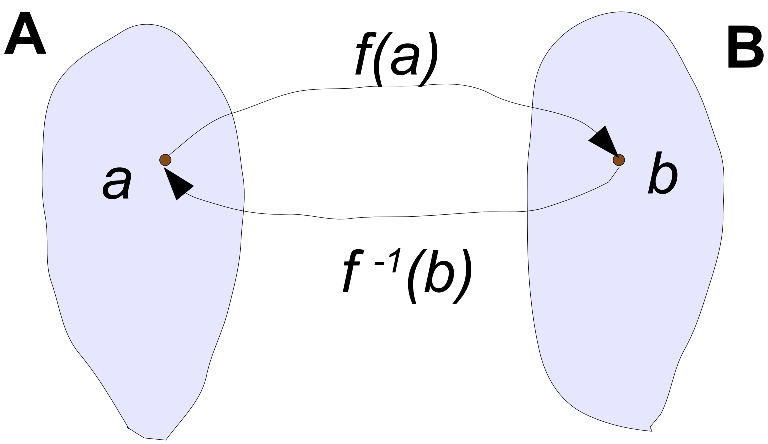
Chapter 3. Functions

3.4 The inverse of a function
3.5 Composition of functions

3.4 The inverse of a function

Let *f* be a one-to-one (injective) function, $f : \mathbf{A} \rightarrow \mathbf{B}$.
The inverse function of *f* is the function that assigns to an element $b \in \mathbf{B}$ the unique element $a \in \mathbf{A}$, such that $f(a) = b$.

denotation: f^{-1} $f^{-1}(b) = a$



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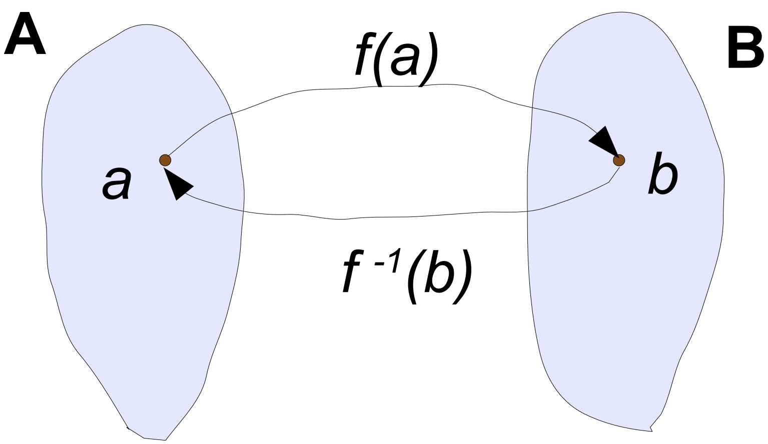

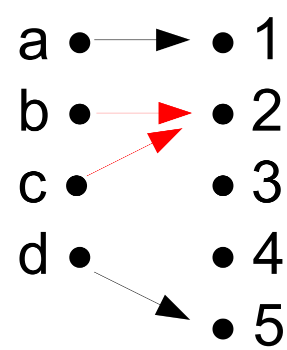

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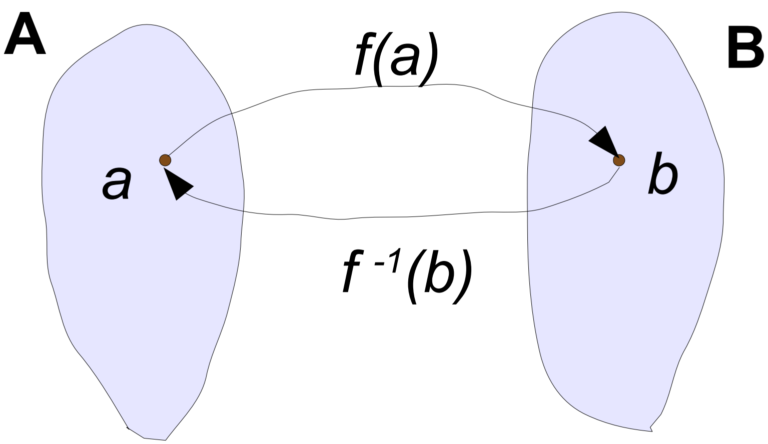

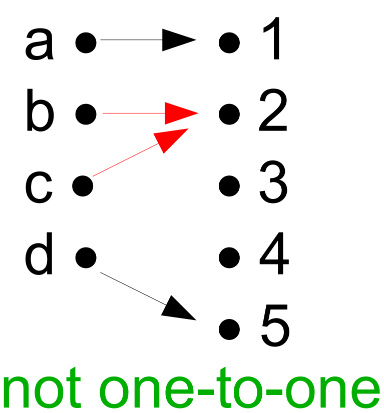

A one-to-one function is invertible because we can define an inverse on it.

A function is not invertible, when we cannot define an inverse (it happens when the function is not one-to-one).

Example 1:
Let *f* be a function, $f:\{a,b,c\} \rightarrow \{1,2,3\}$, with $f(a) = 1$, $f(b) = 3$, $f(c) = 2$. Is *f* invertible? If it is, define its inverse.

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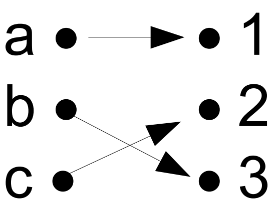

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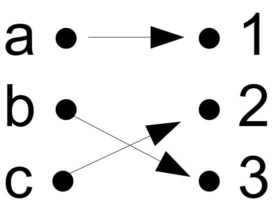

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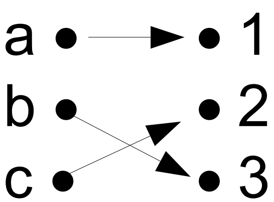

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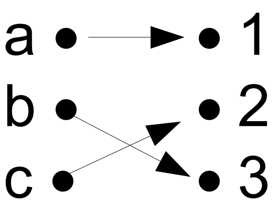

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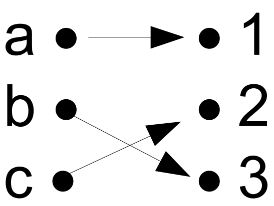

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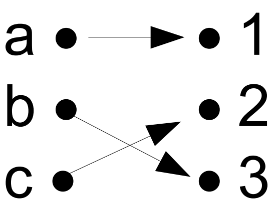

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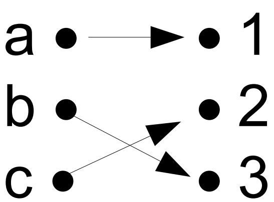

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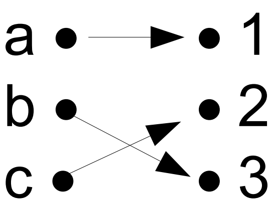

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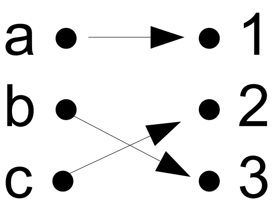

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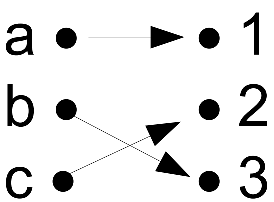

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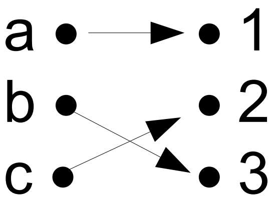

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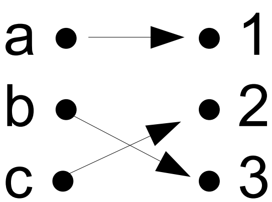

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Example 3:

Let f be a function, $f : \mathbf{R} \to \mathbf{R}$, $f(x) = x^2+2$.

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Can we find two different values x_1 and x_2, such that $f(x_1)=f(x_2)$? Yes
$f(-1) = f(1) = 3$
Therefore, the function is not one-to-one, hence doesn't have an inverse.

More about functions

Let f be a function, f: $\mathbf{R} \to \mathbf{R}$

Function f is increasing if for any $x,y \in \mathbf{R}$, where $x < y$ the following inequality holds: $f(x) \leq f(y)$.

$$\forall x \forall y\ (\ x < y \to f(x) \leq f(y)\ )$$

Function f is strictly increasing if for any $x,y \in \mathbf{R}$, where $x < y$ the following inequality holds: $f(x) < f(y)$.

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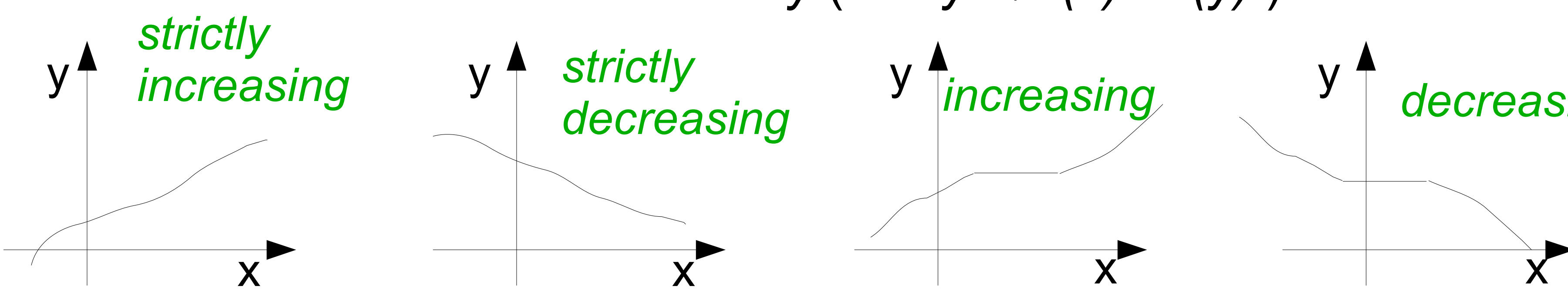

More about functions

Theorem

If function is strictly increasing or strictly decreasing, then it is one-to-one.

However a function that is increasing or decreasing (but not strictly) is not necessary one-to-one.

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Give an example of increasing, strictly increasing, decreasing and strictly decreasing functions.

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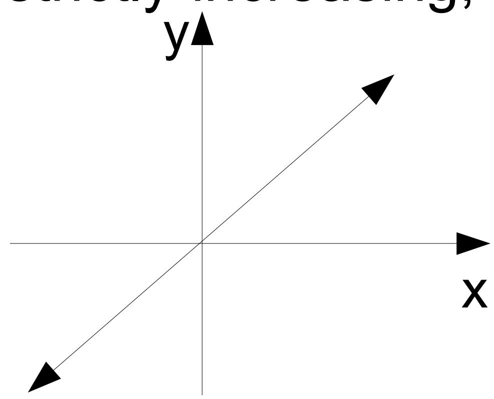

Solution:
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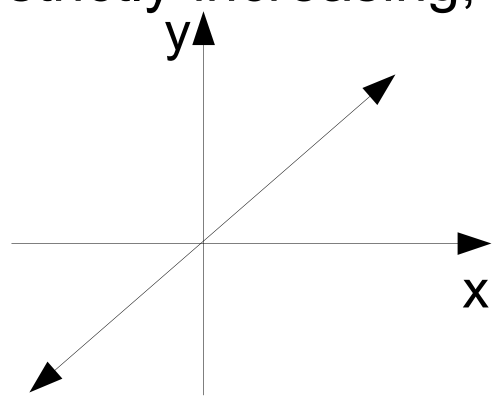


Solution:

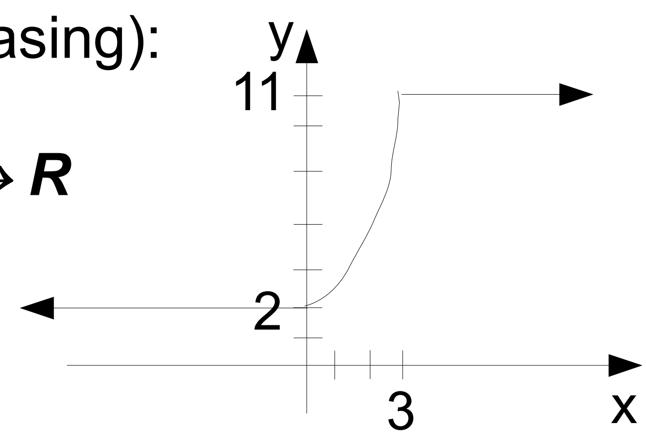
1) strictly increasing function:

$f(x) = x$, f: **R** → **R**

2) increasing function (but not strictly increasing):

$$f(x) = \begin{cases} 2, & \text{if } x < 0 \\ x^2+2, & \text{if } 0 \le x \le 3 \\ 11, & \text{if } x > 3 \end{cases}$$

f: ***R*** → ***R***



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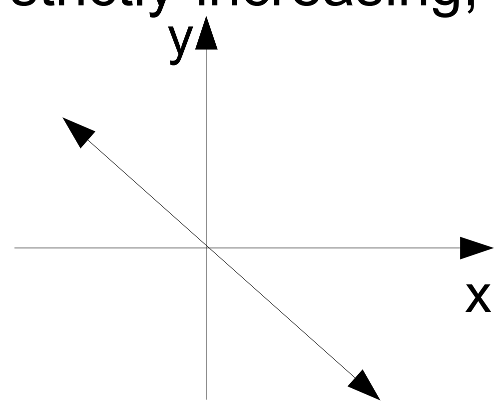
<u>Solution</u>:

3) strictly decreasing function:

$f(x) = -x$, f: $\mathbf{R} \rightarrow \mathbf{R}$



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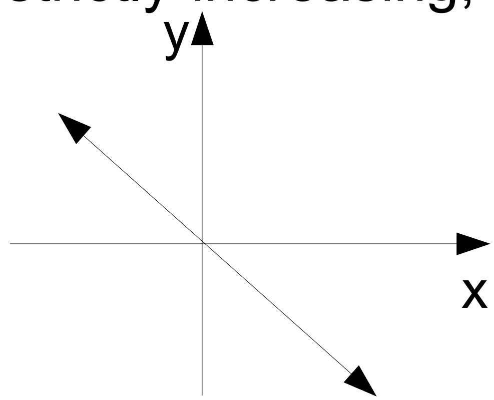
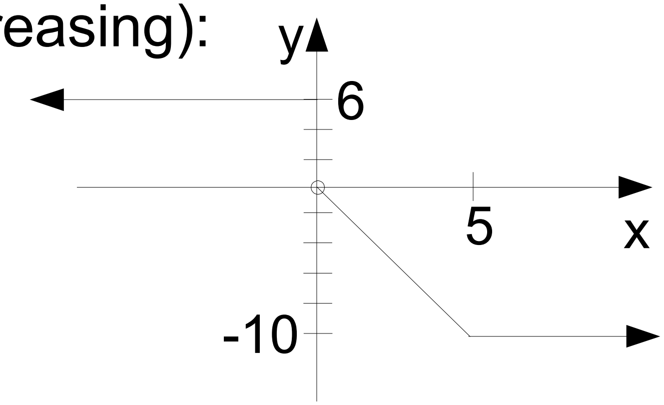


4) decreasing function (but not strictly decreasing):

$$f(x) = \begin{cases} 6, & \text{if } x \le 0 \\ -2x, & \text{if } 0 < x \le 5 \\ -10, & \text{if } x > 5 \end{cases} \qquad f: \boldsymbol{R} \to \boldsymbol{R}$$



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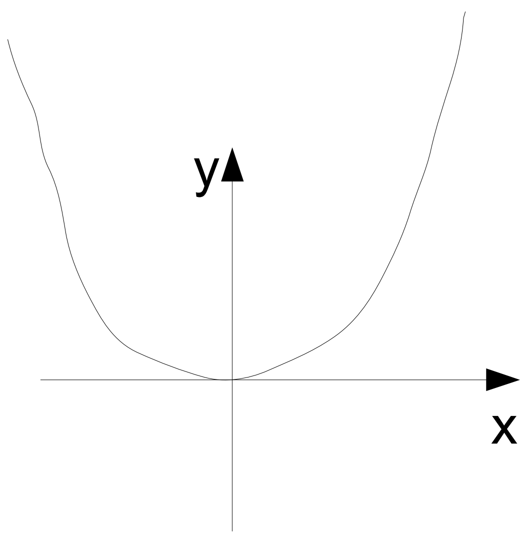
Solution:

5) not increasing, not decreasing:

$f(x) = x^2$, f: $\mathbf{R} \rightarrow \mathbf{R}$



More about functions

Let f_1 and f_2 be functions from A to **R**. Then f_1+f_2 and f_1f_2 are also functions from A to **R**, defined by

$$(f_1+f_2)(x) = f_1(x) + f_2(x)$$
$$(f_1f_2)(x) = f_1(x)f_2(x)$$

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Let $f_1(x)=x^3+3$, $f_1:\mathbf{R} \to \mathbf{R}$ and $f_2(x)=-x+5$, $f_2:\mathbf{R} \to \mathbf{R}$

What are functions f_1+f_2, and f_1f_2 ?

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<u>Solution</u>:
$(f_1+f_2)(x) = (x^3+3) + (-x+5) = x^3-x+8$

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Solution:

$(f_1+f_2)(x) = (x^3+3) + (-x+5) = x^3-x+8$

$(f_1f_2)(x) = (x^3+3)(-x+5) = -x^4+5x^3-3x+15$

Answer: $(f_1+f_2)(x) = x^3-x+8$, *and* $(f_1f_2)(x) = -x^4+5x^3-3x+15$

Composition of Functions

Let f and g be functions, $g : \mathbf{A} \to \mathbf{B}$, $f : \mathbf{B} \to \mathbf{C}$. Then $f \circ g$ is the composition of two functions, defined by $(f \circ g)(x) = f(g(x))$

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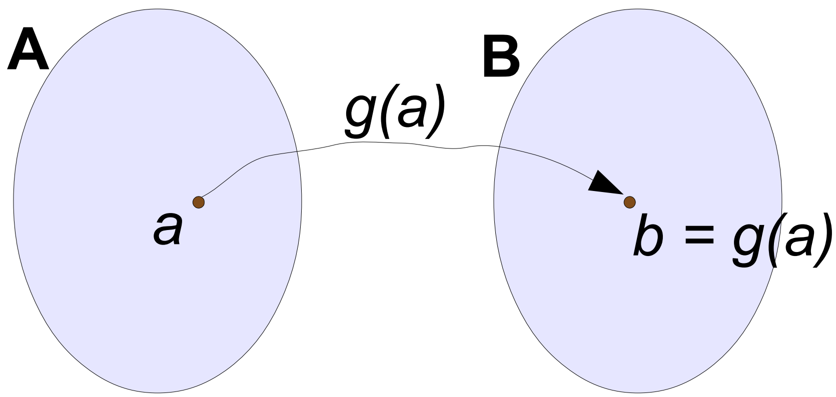

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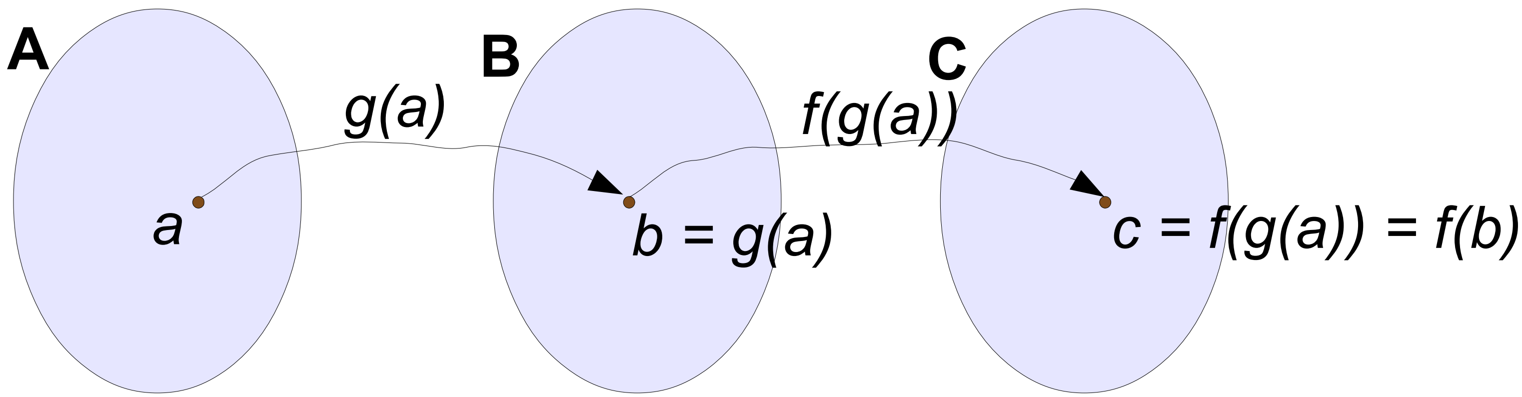

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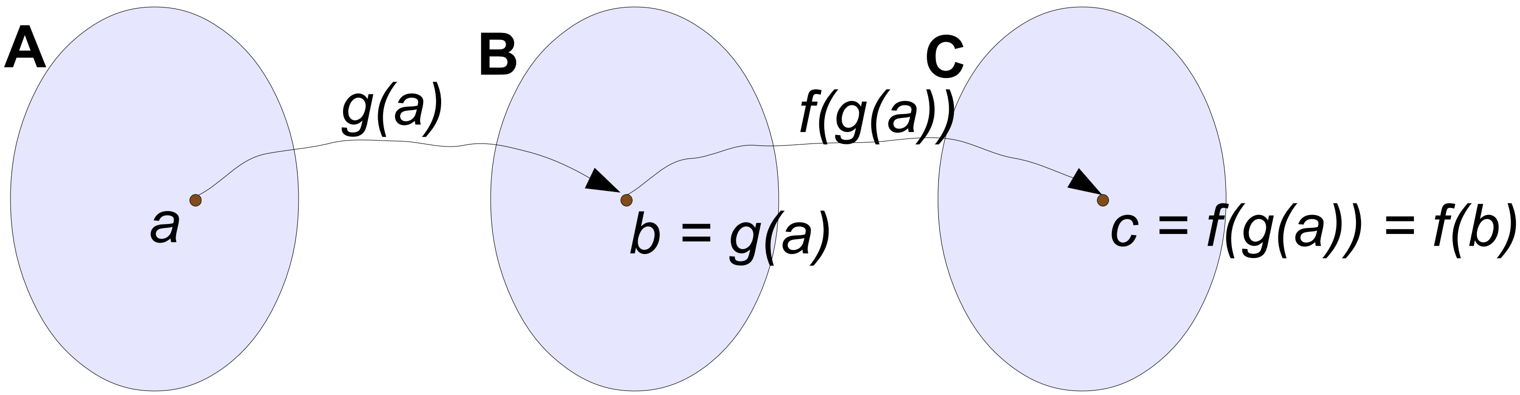

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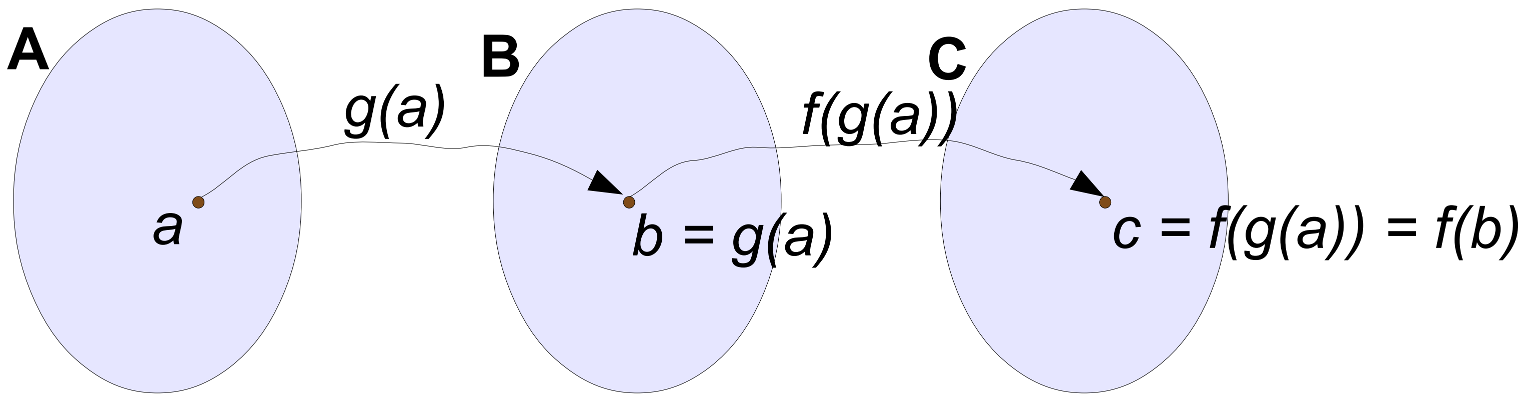

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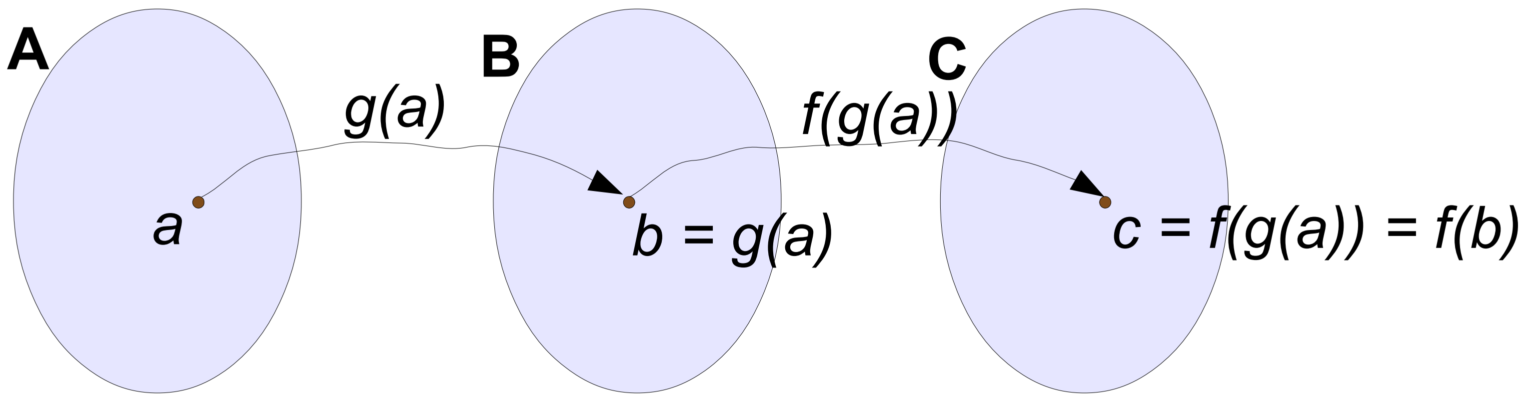

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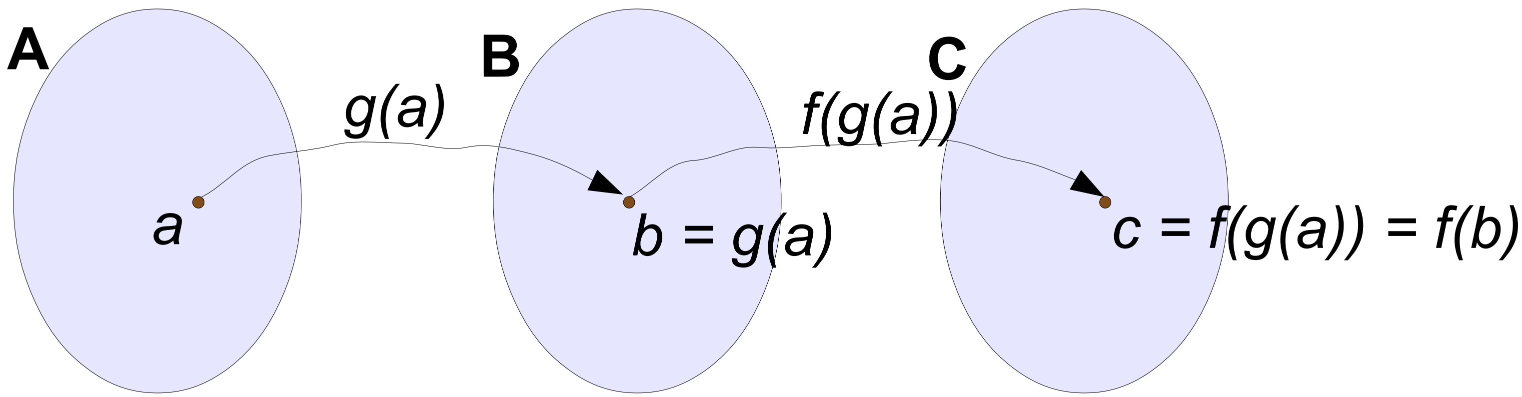

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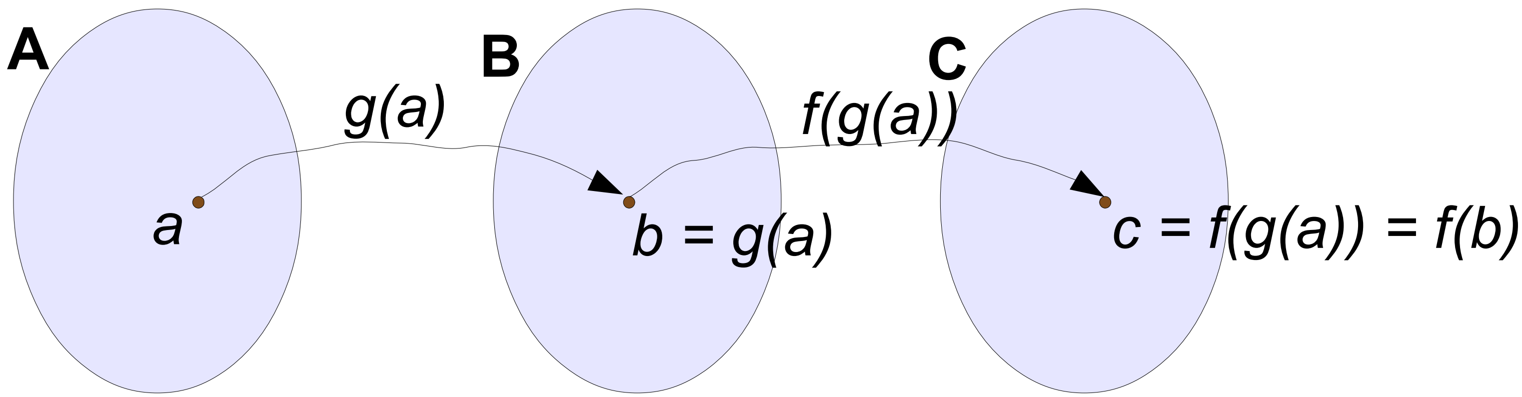

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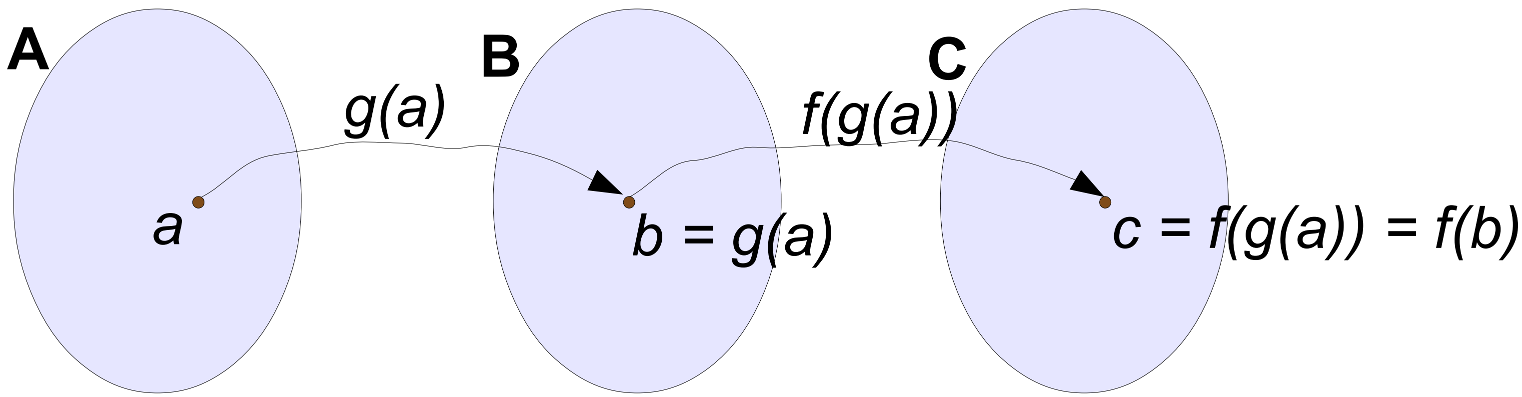

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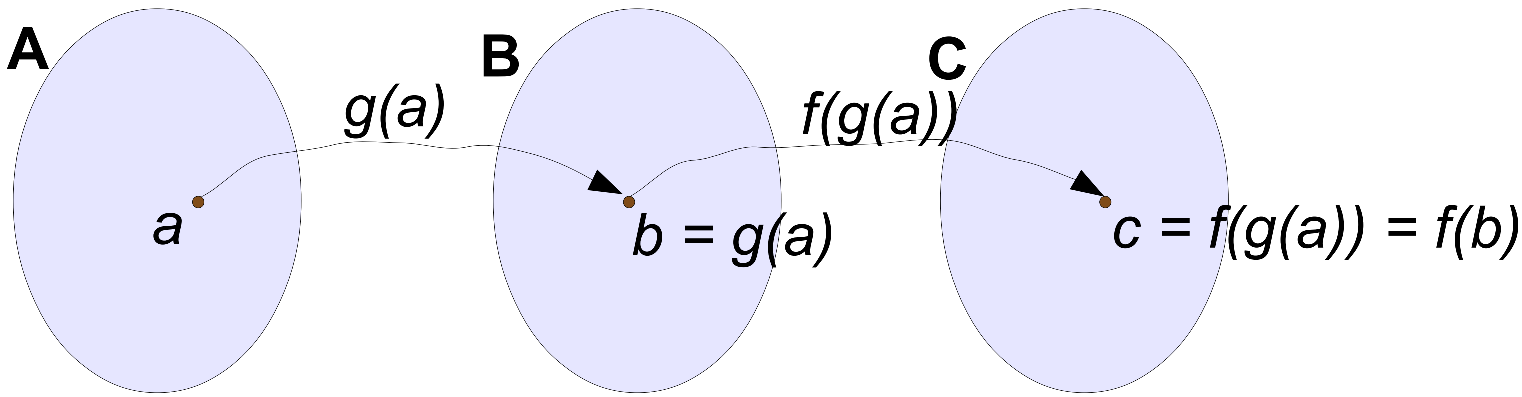

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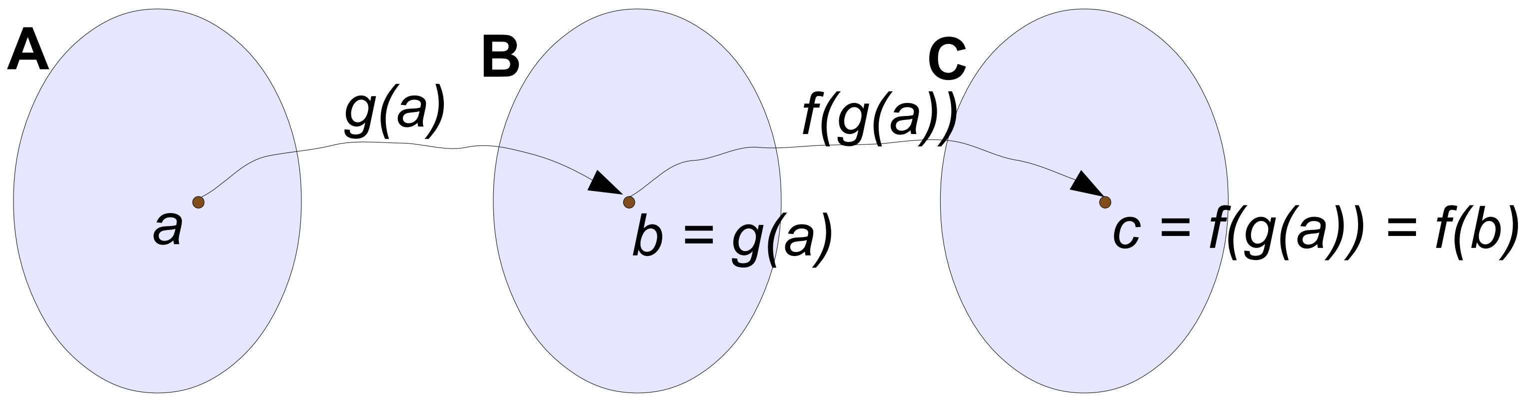

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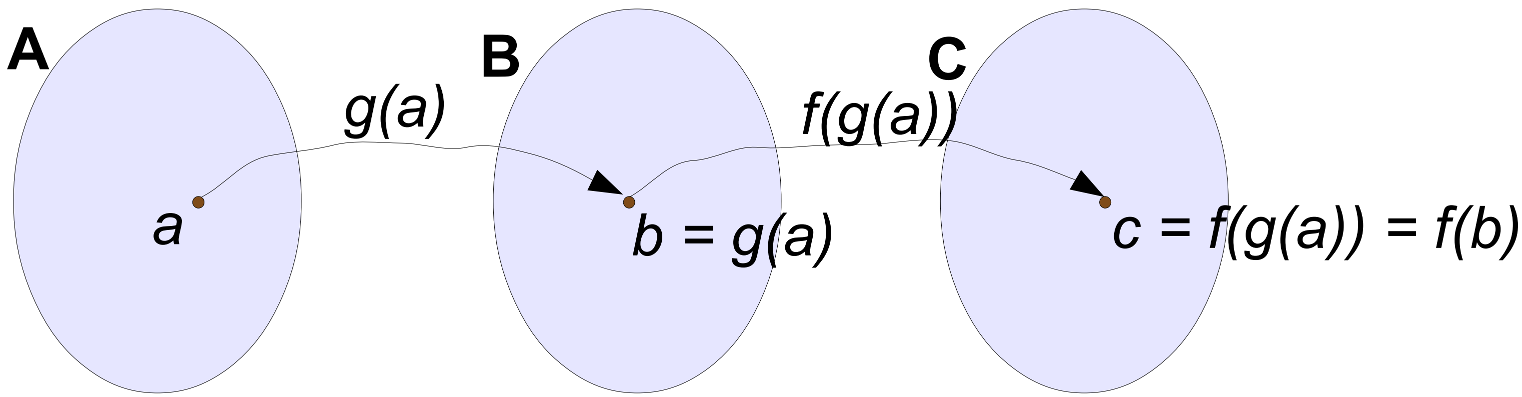

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Let *f* and *g* be functions, $g : \mathbf{Z} \rightarrow \mathbf{Z}$, $f : \mathbf{Z} \rightarrow \mathbf{Z}$.
$f(x) = 3x+5, \quad g(x) = x-3$
Find $f \circ g$ and $g \circ f$.

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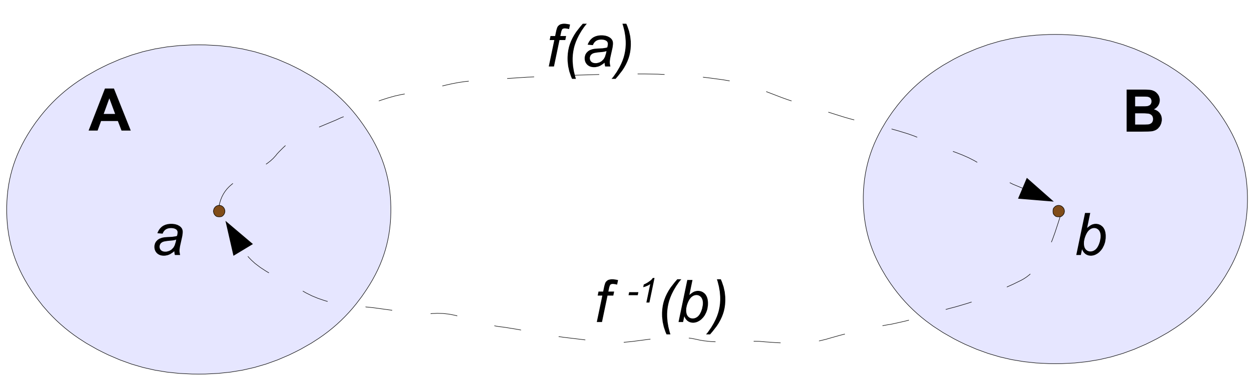

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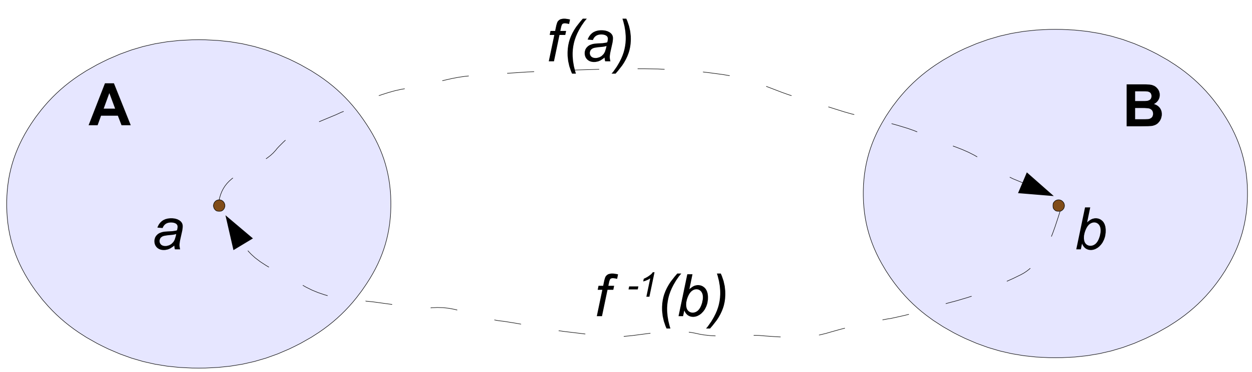

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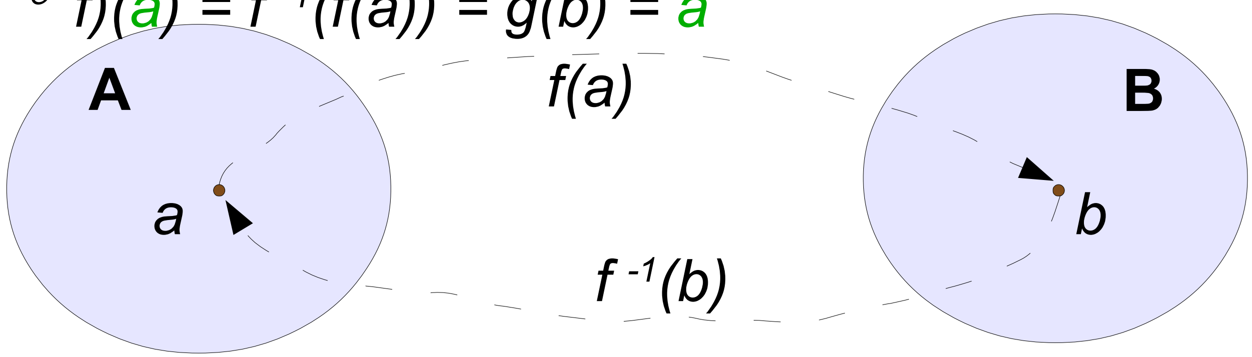

Graphs of Functions

We can associate a set of pairs **A** × **B** to each *function* from **A** to **B**. This set of pairs is called the graph of a function and is often displayed pictorially to aid in understanding the behavior of the function.

Let $f : \mathbf{A} \rightarrow \mathbf{B}$. The graph of the function is the set of ordered pairs $\{ (a,b) \mid a \in \mathbf{A} \text{ and } f(a) = b \}$

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Display the graphs of the given functions
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x	*3x-3*
0	
1	
2	
-1	
-2	

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x	*3x-3*
0	-3
1	0
2	3
-1	-6
-2	-9

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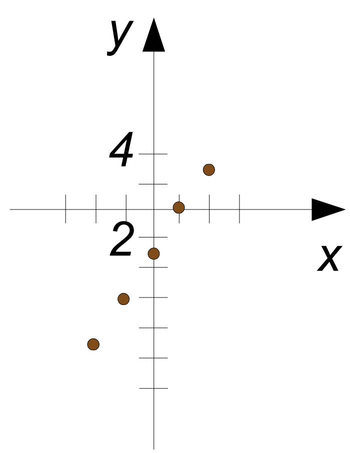
1	0
2	3
-1	-6
-2	-9



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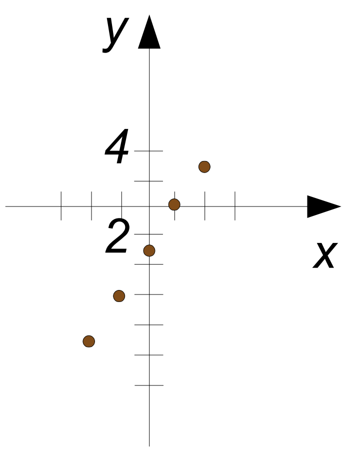

b) $f(x) = x^2+2$

x	x^2+2
0	
1	
2	
-1	
-2	

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0	-3
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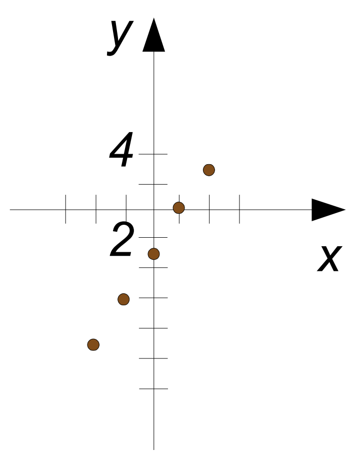
2	3
-1	-6
-2	-9



b) $f(x) = x^2+2$

x	x^2+2
0	2
1	3
2	6
-1	3
-2	6

Graphs of Functions

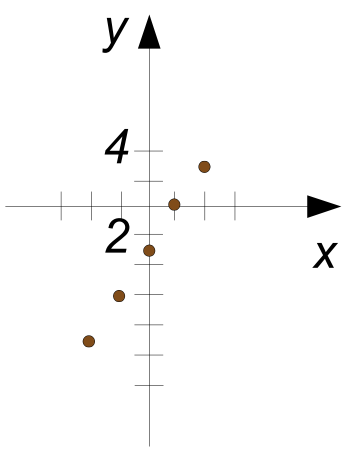
We can associate a set of pairs **A** × **B** to each *function* from **A** to **B**. This set of pairs is called the graph of a function and is often displayed pictorially to aid in understanding the behavior of the function.

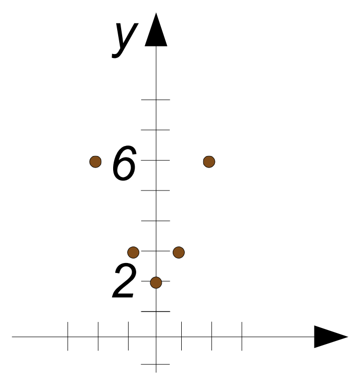
Let *f* : **A** → **B**. The graph of the function is the set of ordered pairs $\{ (a,b) \mid a \in$ **A** and $f(a) = b\}$

Example 7:
Display the graphs of the given functions
a) $f(x) = 3x-3$, *f*: **Z** → **Z**
b) $f(x) = x^2+2$, f: **Z** → **Z**

<u>Solution</u>:
a) $f(x) = 3x-3$

x	$3x-3$
0	-3
1	0
2	3
-1	-6
-2	-9





b) $f(x) = x^2+2$

x	x^2+2
0	2
1	3
2	6
-1	3
-2	6