Chapter 3. Functions

3.1 Definition of functions
3.2 Floor and ceiling functions
3.3 Properties of functions

3.1 Definition of functions

$y = f(x)$ $y = x^2+10$ $f(x) = \sin(x) + \cos(x)$

Let **A**, **B** be non-empty sets. A function *f* from **A** to **B** is an assignment of exactly one element of **B** to each element of **A**.

denotations:

$f(a) = b$ (if *b* is unique element from **B** assigned to *a* from **A** by *f*)

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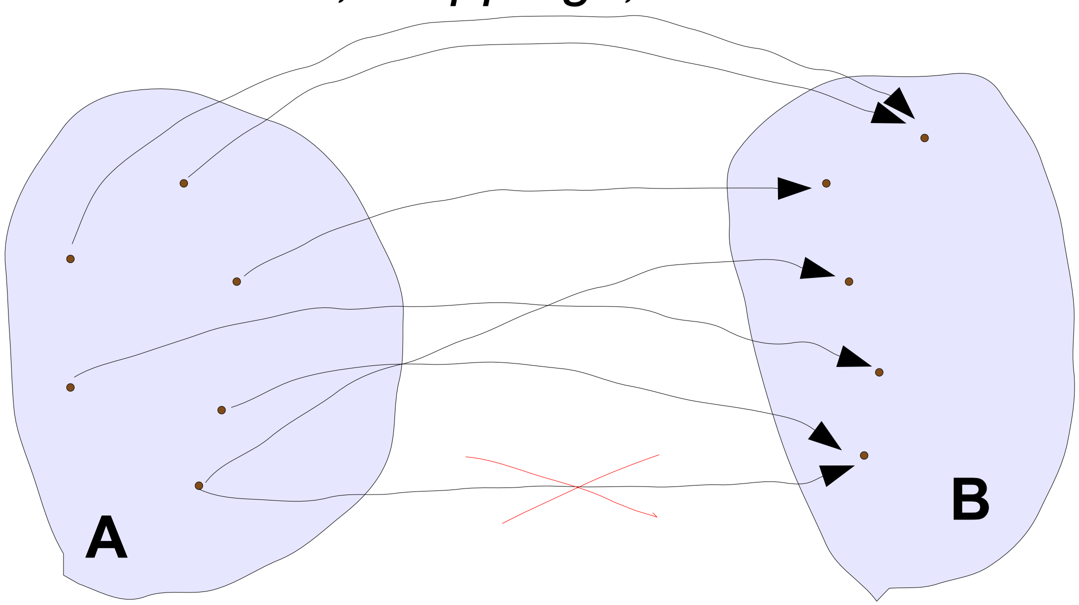

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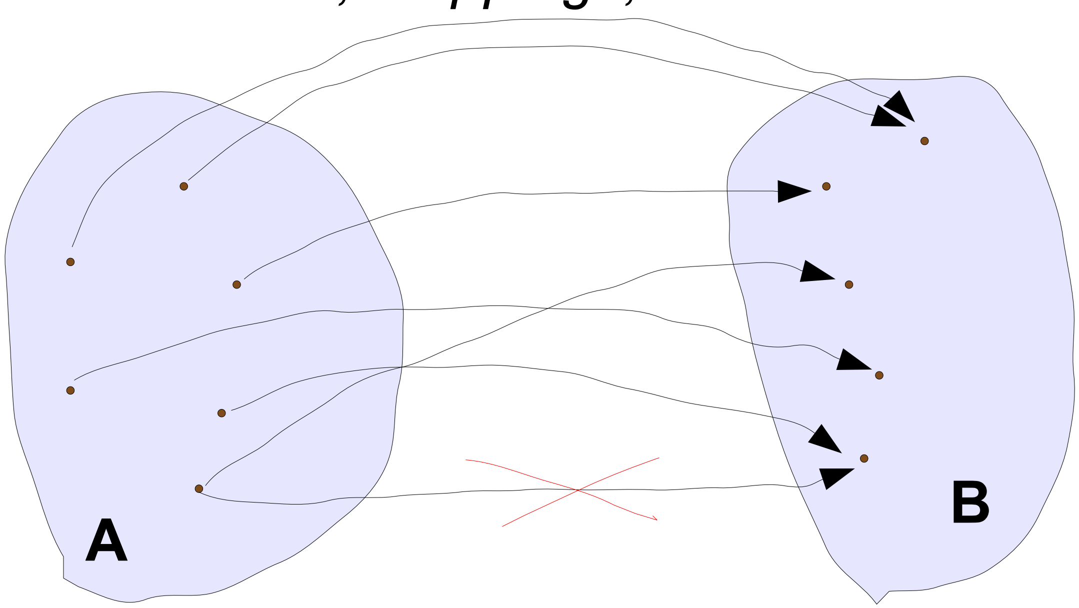
target



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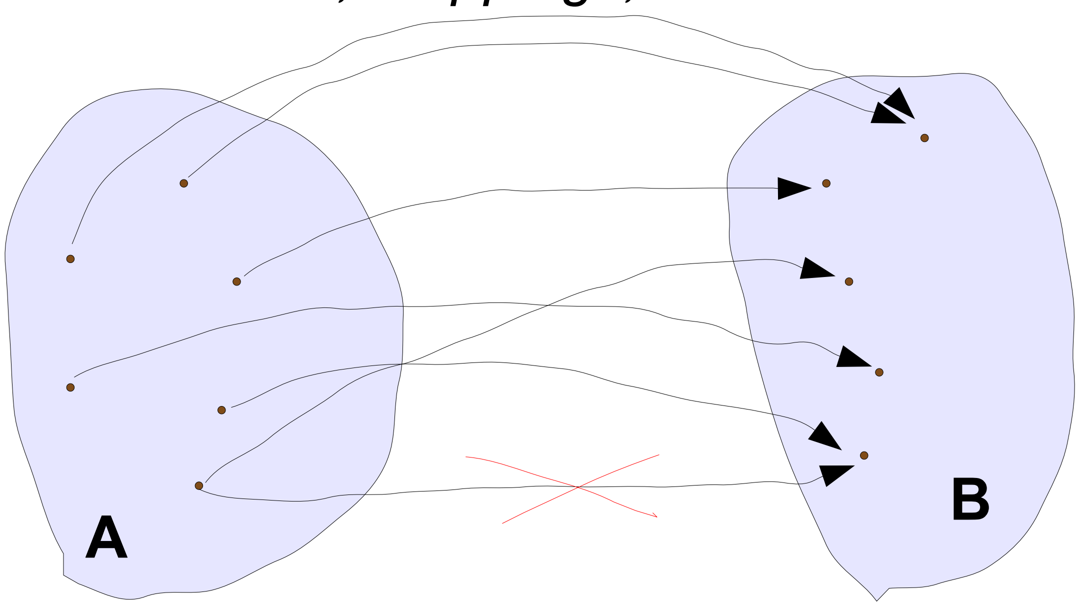

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if *f(a) = b*, then
b is the *image* of a,
a is a *preimage* of b

range of f is the set of all images of elements of A

3.1 Definition of functions

there is another way to define a function:

A relation from **A** to **B** is a subset of **A** $\times$ **B**. <u>denotation</u>: R

A relation from **A** to **B** that contains one and only one ordered pair *(a,b)* for every element $a \in$ **A**, defines a function *f* from **A** to **B**.

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R is the relation consisting of pairs: (Tom,22), (Elsa,30), (Maria,40), and (Kevin,30), where each pair consists of the person's name and the age of the person.
Can we define a function?
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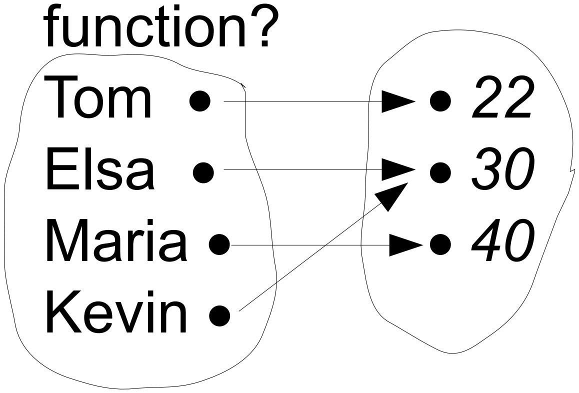

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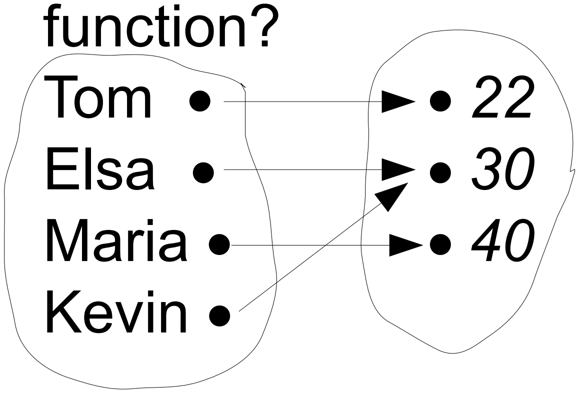

Yes, this relation defines a function *f*, where *f*(Tom) = *22*, *f*(Elsa) = *30*, *f*(Maria) = *40*, and *f*(Kevin) = *30*.

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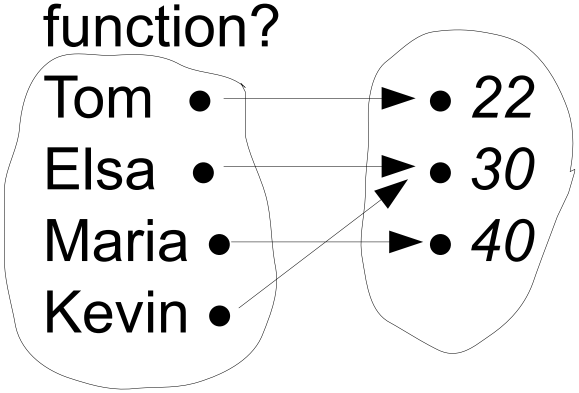
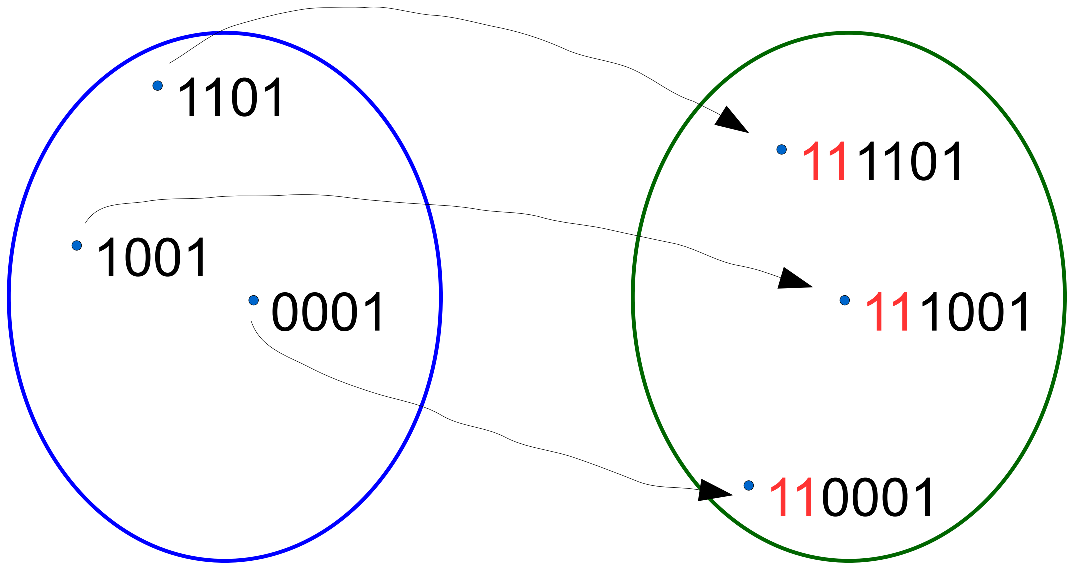


Yes, this relation defines a function *f*, where *f*(Tom) = *22*, *f*(Elsa) = *30*, *f*(Maria) = *40*, and *f*(Kevin) = *30*.
domain: {Tom, Elsa, Maria, Kevin}
Codomain/target: positive integers < *140*
range: {*22*,*30*,*40*}

3.1 Definition of functions

Example 2:

Let's define $f : \{0,1\}^4 \rightarrow \{0, 1\}^6$ such that for $x \in \{0,1\}^4$, $f(x) = 11x$



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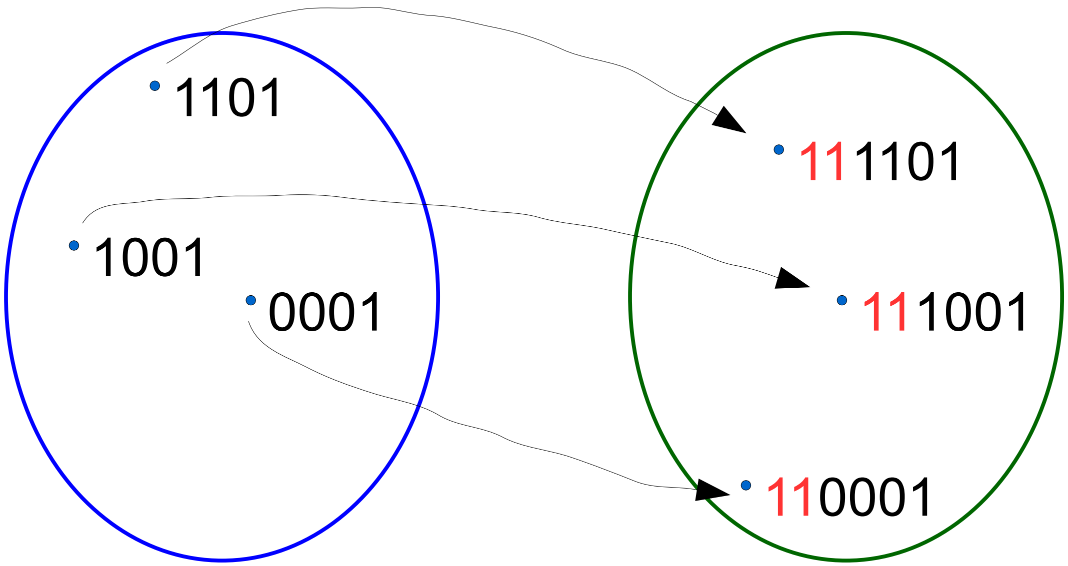

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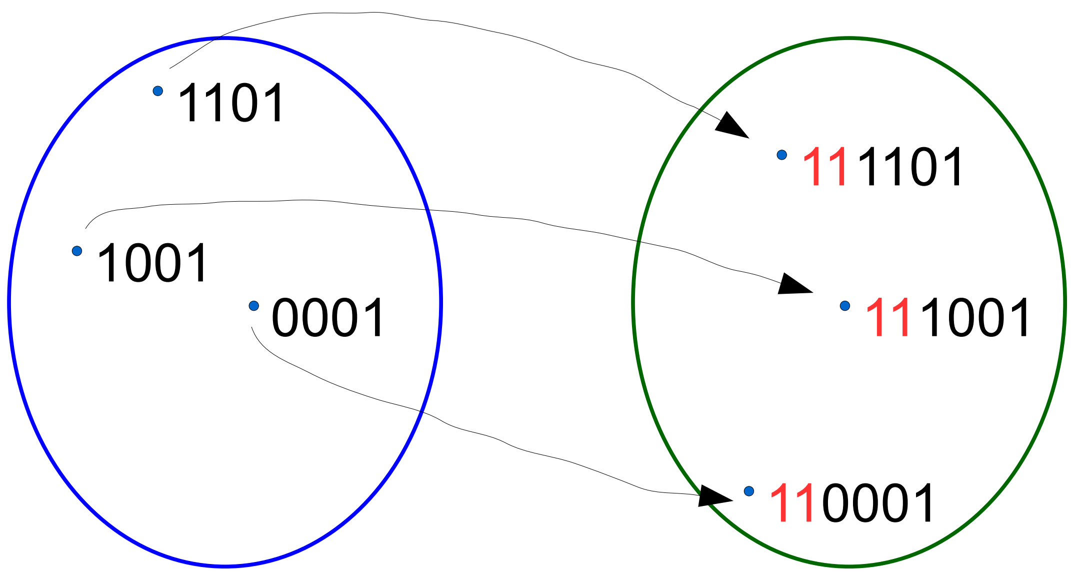

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Codomain/target = all bit strings of length 6

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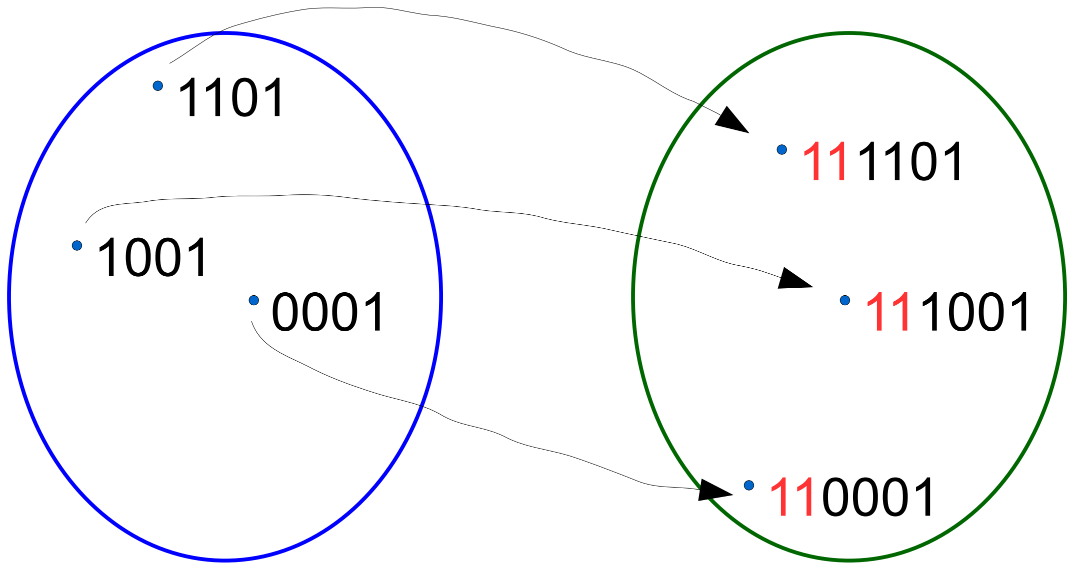

domain = all bit strings of length 4

Codomain/target = all bit strings of length 6

range = all bit strings of length 6 that start with 11

Practice Problems

1. R is the relation consisting of pairs: (Tom,22), (Elsa,30), (Maria,40), and (Kevin,30), (Tom,54) where each pair consists of the person's name and the age of the person.

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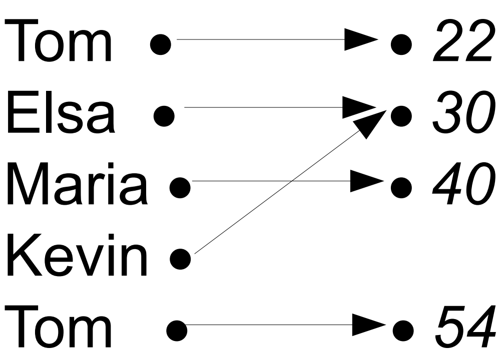

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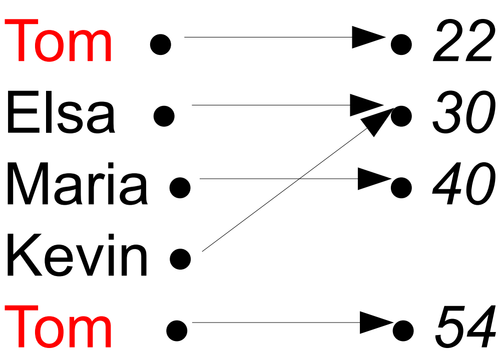

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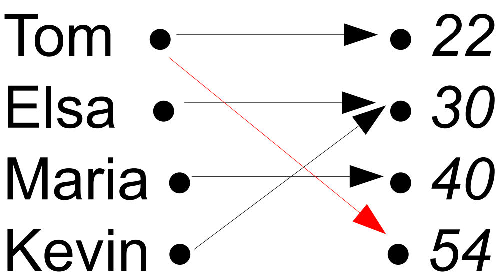

not a function

Practice Problems

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a) the function that assigns to a bit string the number of bits in the string

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range: $\mathbf{Z}^+$ (positive integers)

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f(1) = 1, f(2) = 1, f(3) = 1, f(4)=4, f(5)=4, f(6) = 4, f(7) = 4, f(8) = 4, f(9) = 9..

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Two functions are equal when they have the same domain, codomain/target and map the elements of their common domain to the same elements in their common codomain/target.

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Example 1:

f: **R** → **R** $\quad f(x) = 2x^2-4x$
g: **R** → **R** $\quad g(x) = 2(x^2-2)$

$f(x)$ and $g(x)$ are equal.

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f(x) and *g(x)* are equal.

Example 2:

f: {a,b,c} → {1,2,3} f(a) = 1, f(b) = 2, f(c) = 3
g: {a,b,c} → {1,2,3}
g(x) = "index of the letter x in the ordered tuple (a,b,c)"

f(x) and g(x) are equal

3.2 Floor and ceiling functions

the floor function assigns to the real number *x* the largest integer that is less than or equal to *x*.

denotation: $\lfloor x \rfloor$

the ceiling function assigns to the real number *x* the smallest integer that is greater than or equal to *x*.

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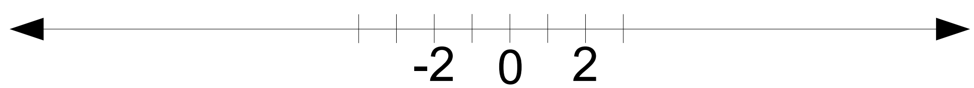
a) $\lfloor \frac{3}{4} \rfloor$



b) $\lceil 0.5 \rceil$

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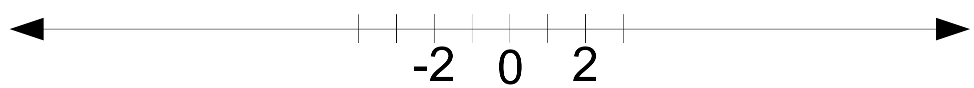

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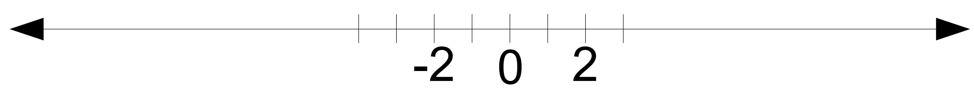

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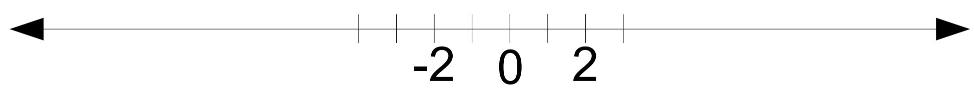

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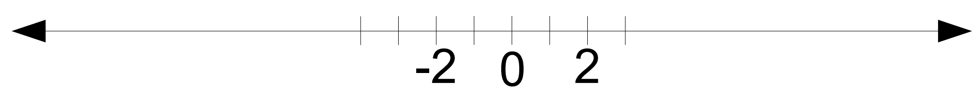

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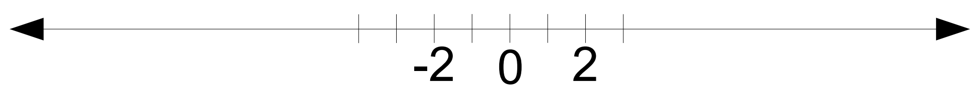

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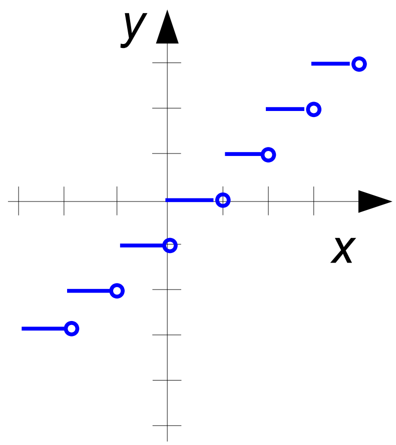
Let's draw the graphs of the floor and ceiling functions, $f(x) = \lfloor x \rfloor$, $g(x) = \lceil x \rceil$



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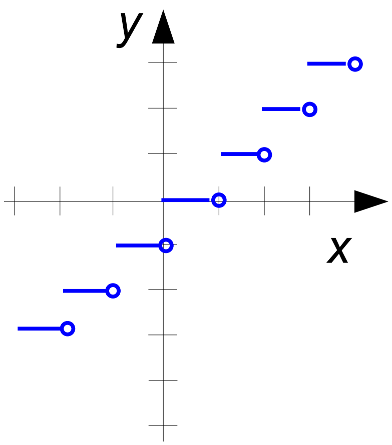

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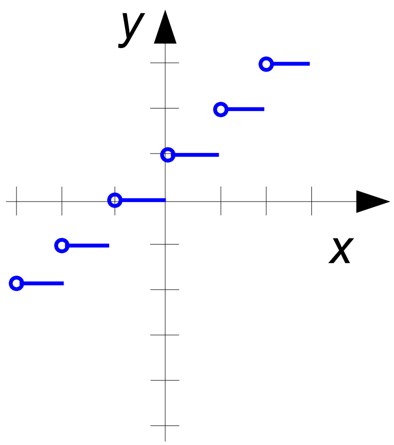

$g(x) = \lceil x \rceil$

Factorial Function

the factorial function $f : \mathbf{N} \rightarrow \mathbf{Z}^+$

$f(n) = n!$

one of the definitions: $0! = 1$ $1! = 1$ $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

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Example 5: Find

a) 3!

b) 7!

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one of the definitions: $0! = 1$ $1! = 1$ $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

Example 5: Find
a) $3! = 1 \cdot 2 \cdot 3$
b) $7!$

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Example 5: Find

a) $3! = 1 \cdot 2 \cdot 3 = 6$

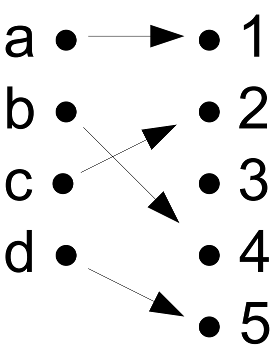
b) $7! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 = 5040$

3.3 Properties of functions

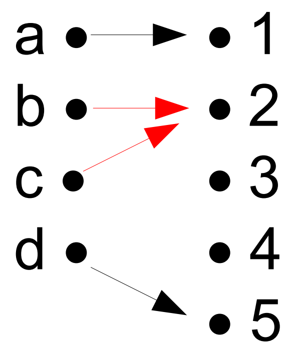
Let f be a function.

If f never assigns the same value to two different domain elements, then it is called one-to-one or injective, i.e.
$f(a) \neq f(b)$ for all a and b, such that $a \neq b$; or

$f(a) = f(b)$ implies that $a=b$



one-to-one



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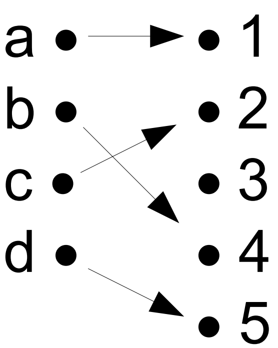

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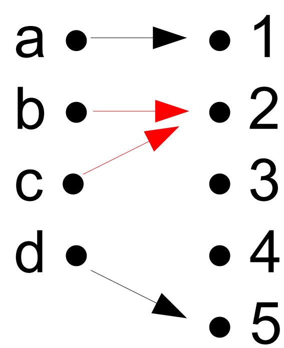

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determine whether the given functions are one-to-one

a) $f(x) = x^3$; f: $\mathbf{Z} \rightarrow \mathbf{Z}$

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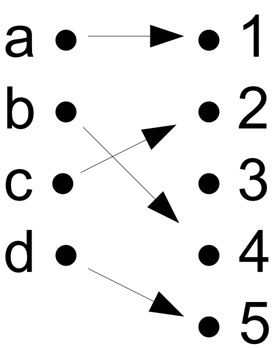

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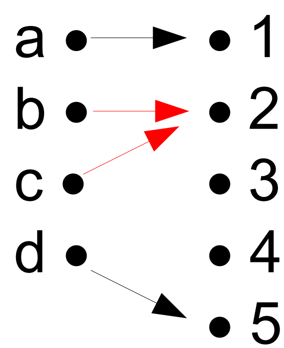

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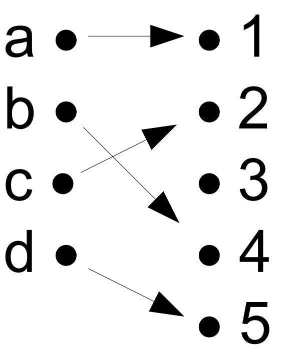

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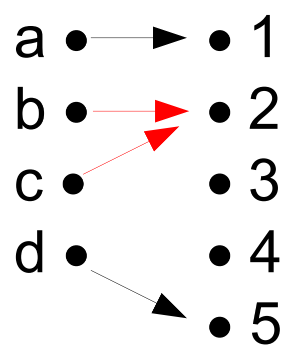

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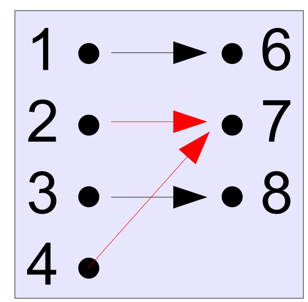

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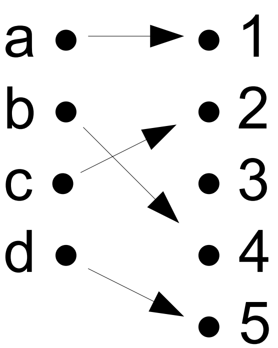

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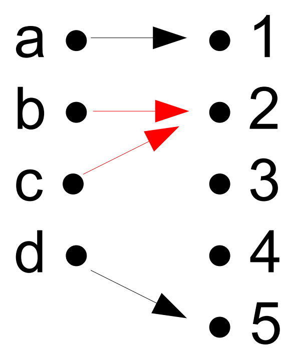

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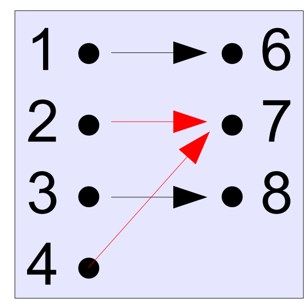

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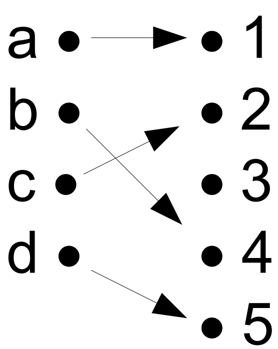

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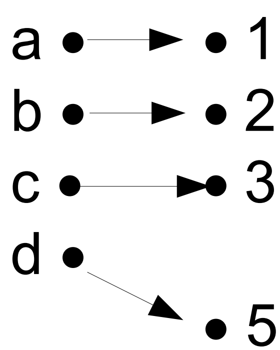

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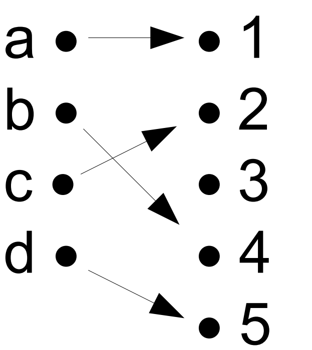

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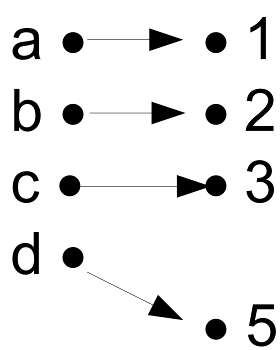

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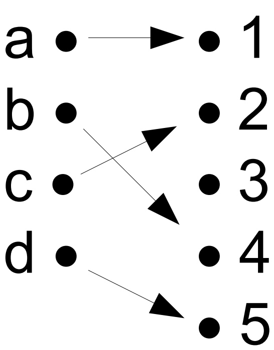

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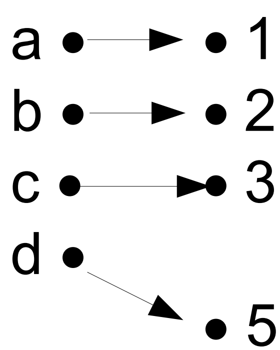

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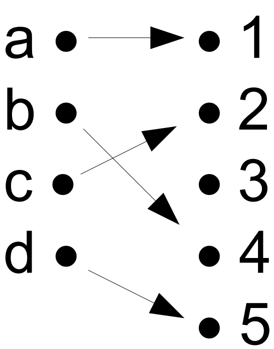

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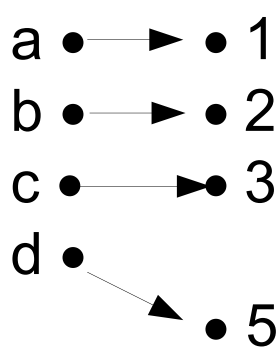

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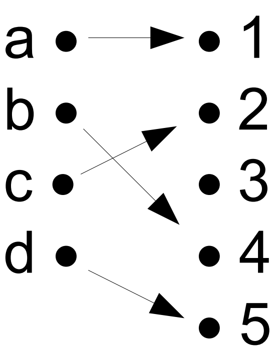

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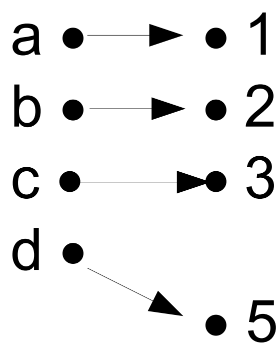

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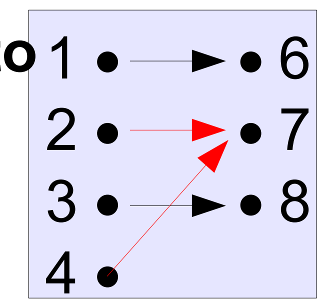

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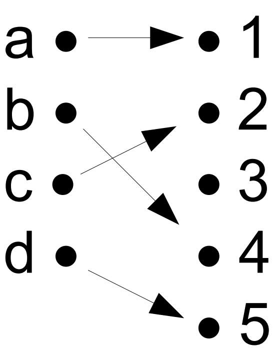

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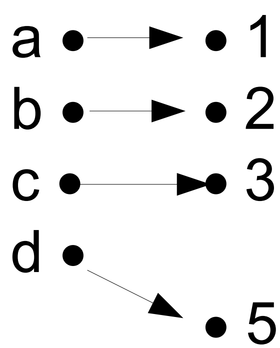

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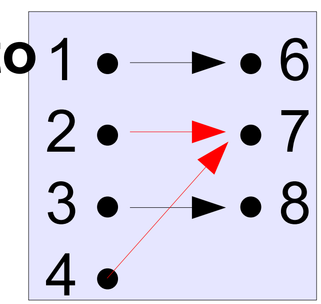

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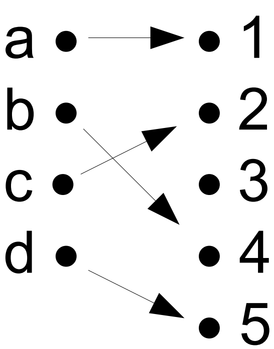

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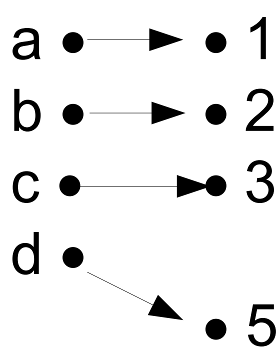

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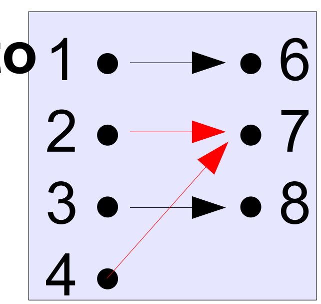

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Let f:{$1,2,3,4,5$} $\rightarrow$ {$6,7,8,9,10$}, with $f(1) = 10$, $f(2) = 9$, $f(3) = 8$, $f(4) = 7$, $f(5) = 6$. Is f bijective?

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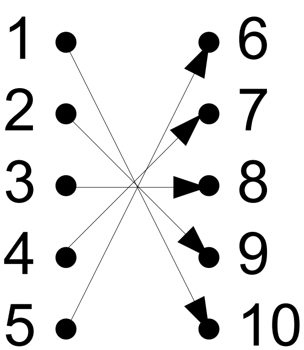

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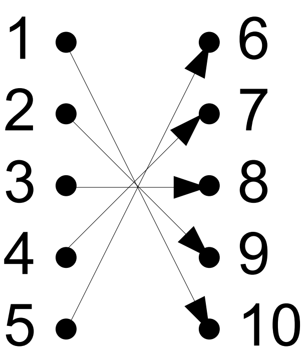

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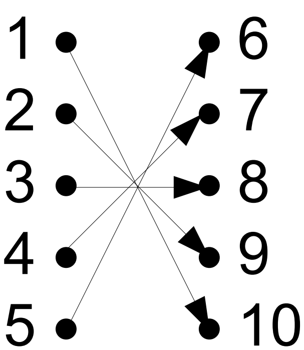

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We also can see that no two different elements from the domain have the same image, therefore it is one-to-one.

The given function is bijective.

Practice Problems

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$f(1) = 1, f(2) = 1, f(3) = 1, f(4)=4, f(5)=4, f(6) = 4, f(7) = 4, f(8) = 4, f(9) = 9..$

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(one-to-one) Can two different bit strings have the same "length"? Yes
therefore it is not one-to-one

(onto) for any value n from the range (positive integer) can we find a bit string that has n bits? Yes
therefore it is onto

b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

$f(1) = 1$, $f(2) = 1$, $f(3) = 1$, $f(4)=4$, $f(5)=4$, $f(6) = 4$, $f(7) = 4$, $f(8) = 4$, $f(9) = 9$..

not one-to-one, because of $f(4)=4$, $f(5)=4$, for example

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

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b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

$f(1) = 1, f(2) = 1, f(3) = 1, f(4)=4, f(5)=4, f(6) = 4, f(7) = 4, f(8) = 4, f(9) = 9..$

not one-to-one, because of $f(4)=4, f(5)=4$, for example
not onto, because *2* is from the *codomain/target*, but not from the *range* of *f*

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

c) $f(n) = n-1$; $f: \mathbf{Z} \to \mathbf{Z}$

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

c) $f(n) = n-1$; $f: \mathbf{Z} \to \mathbf{Z}$

f(-2) *= -2-1=**-3**, **f(-1)** = -1-1=**-2**, **f(0)** = 0-1=**-1**, **f(1)** = 1-1=**0**, **f(2)** = 2-1=**1***

one-to-one, because for two different values from the domain there are two different images;

Practice Problems

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one-to-one, because for two different values from the domain there are two different images;
onto, because for any value *s* from the codomain/target, there is a value in the domain: $(s+1)$ $f(s+1) = s$
therefore, the given function is bijective