Chapter 3. Functions

3.1 Definition of functions
3.2 Floor and ceiling functions
3.3 Properties of functions

3.1 Definition of functions

$y = f(x)$ $y = x^2+10$ $f(x) = \sin(x) + \cos(x)$

Let **A**, **B** be non-empty sets. A function *f* from **A** to **B** is an assignment of exactly one element of **B** to each element of **A**.

denotations:

$f(a) = b$ (if *b* is unique element from **B** assigned to *a* from **A** by *f*)

$f: A \to B$ f is a function from A to B

3.1 Definition of functions

$y = f(x)$ $y = x^2+10$ $f(x) = \sin(x) + \cos(x)$

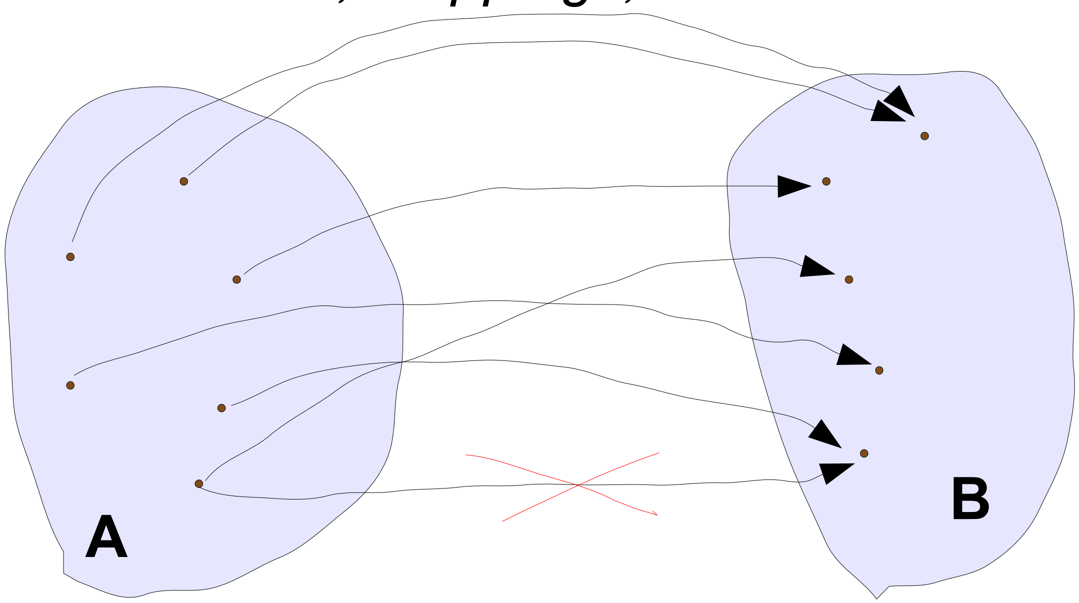
Let **A**, **B** be non-empty sets. A function *f* from **A** to **B** is an assignment of exactly one element of **B** to each element of **A**.

<u>denotations</u>:

$f(a) = b$ (if *b* is unique element from **B** assigned to *a* from **A** by *f*)

$f: A \to B$ f is a function from A to B

names: *functions, mappings, transformations*



f maps **A** to **B** (*f* is a function from **A** to **B**)

3.1 Definition of functions

y = f(x) *y = x*^{2}*+10* *f(x) = sin(x) + cos(x)*

Let **A**, **B** be non-empty sets. A function *f* from **A** to **B** is an assignment of exactly one element of **B** to each element of **A**.

denotations:

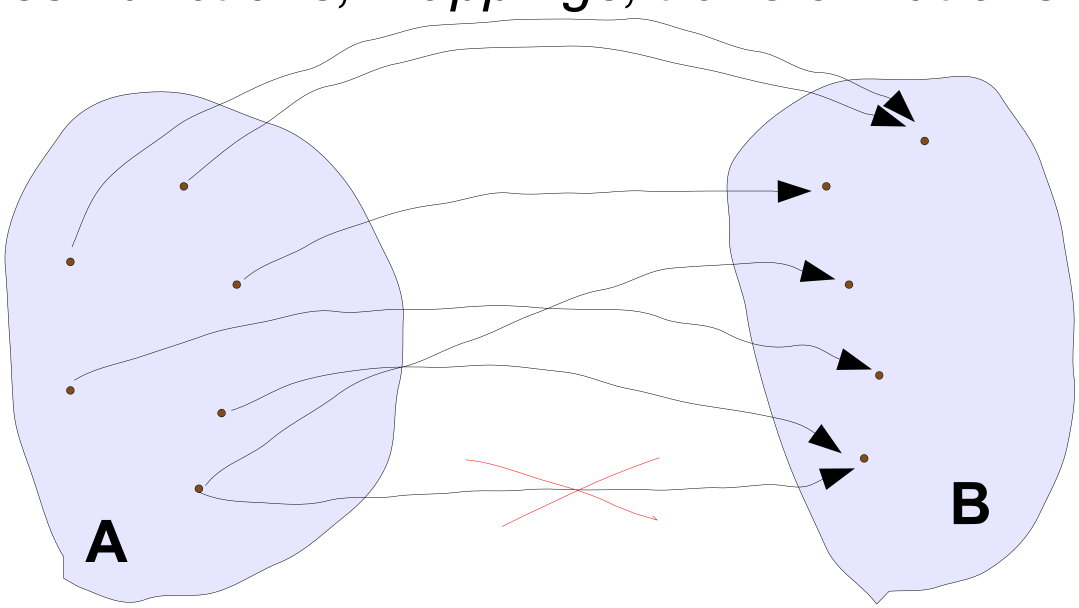
f(a) = b (if *b* is unique element from **B** assigned to *a* from **A** by *f*)

f: A $\rightarrow$ *B* f is a function from A to B

names: *functions, mappings, transformations*

domain: set **A**

codomain: set **B**
target



f maps **A** to **B** (*f* is a function from **A** to **B**)

3.1 Definition of functions

$y = f(x)$ $y = x^2+10$ $f(x) = \sin(x) + \cos(x)$

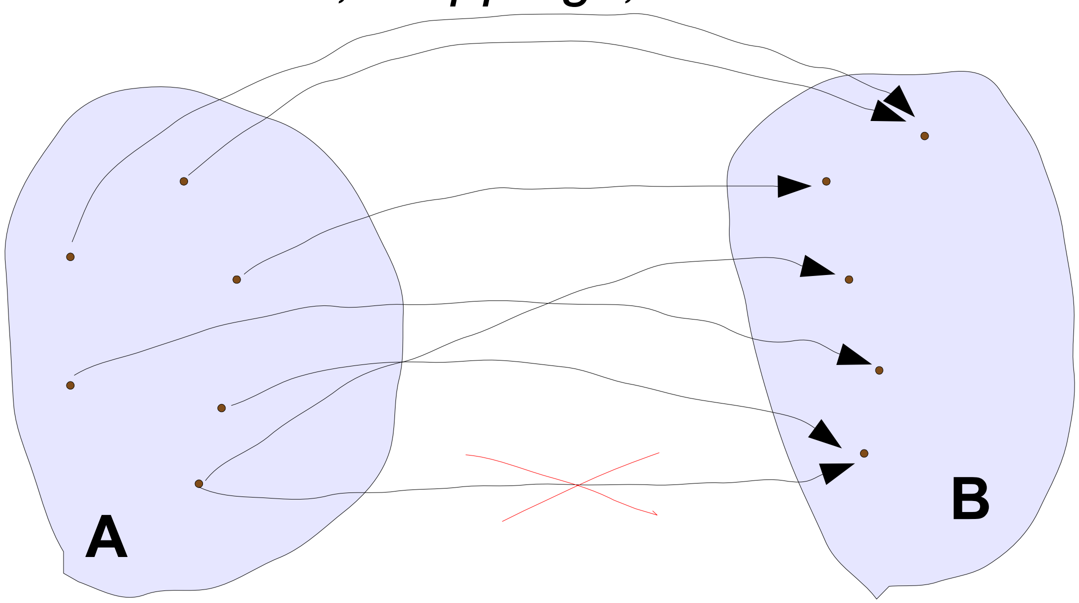
Let **A**, **B** be non-empty sets. A function *f* from **A** to **B** is an assignment for exactly one element of **B** to each element of **A**.

<u>denotations</u>:

$f(a) = b$ (if *b* is unique element from **B** assigned to *a* from **A** by *f*)

$f: A \rightarrow B$ f is a function from A to B

names: *functions, mappings, transformations*



f maps **A** to **B** (*f* is a function from **A** to **B**)

domain: set **A**

codomain: set **B** (target)

if $f(a) = b$, then b is the *image* of a, a is a *preimage* of b

range of f is the set of all images of elements of A

3.1 Definition of functions

there is another way to define a function:

A relation from **A** to **B** is a subset of **A** $\times$ **B**. <u>denotation</u>: R

A relation from **A** to **B** that contains one and only one ordered pair *(a,b)* for every element $a \in$ **A**, defines a function *f* from **A** to **B**.

This function is defined by the assignment *f(a)* = *b*, where *(a,b)* is the unique ordered pair that has *a* as its first element.

3.1 Definition of functions

there is another way to define a function:

A relation from **A** to **B** is a subset of **A** $\times$ **B**. denotation: R

A relation from **A** to **B** that contains one and only one ordered pair *(a,b)* for every element $a \in$ **A**, defines a function *f* from **A** to **B**.

This function is defined by the assignment $f(a) = b$, where *(a,b)* is the unique ordered pair that has *a* as its first element.

Example 1:
R is the relation consisting of pairs: (Tom,22), (Elsa,30), (Maria,40), and (Kevin,30), where each pair consists of the person's name and the age of the person.
Can we define a function?
If we can, what is the domain, the codomain/target, and the range of this function?

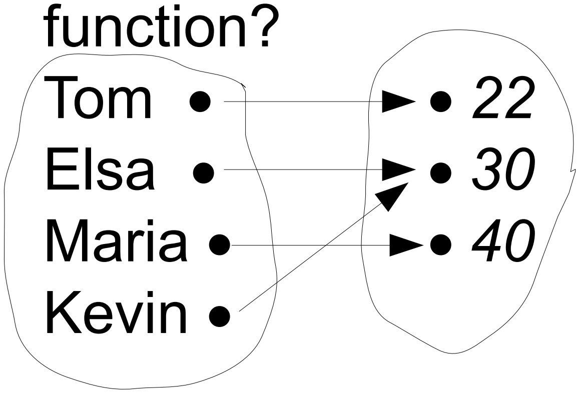
there is another way to define a function:

A relation from **A** to **B** is a subset of **A** $\times$ **B**. <u>denotation</u>: R

A relation from **A** to **B** that contains one and only one ordered pair *(a,b)* for every element $a \in$ **A**, defines a function *f* from **A** to **B**.

This function is defined by the assignment $f(a) = b$, where *(a,b)* is the unique ordered pair that has *a* as its first element.

Example 1:
R is the relation consisting of pairs: (Tom,22), (Elsa,30), (Maria,40), and (Kevin,30), where each pair consists of the person's name and the age of the person.
Can we define a function?
If we can, what is the domain, the codomain/target, and the range of this function?



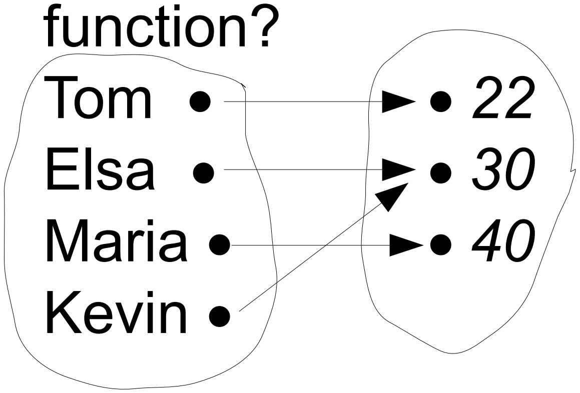
there is another way to define a function:

A relation from **A** to **B** is a subset of **A** $\times$ **B**. <u>denotation</u>: R

A relation from **A** to **B** that contains one and only one ordered pair *(a,b)* for every element $a \in$ **A**, defines a function *f* from **A** to **B**.

This function is defined by the assignment *f(a)* = *b*, where *(a,b)* is the unique ordered pair that has *a* as its first element.

Example 1:
R is the relation consisting of pairs: (Tom,22), (Elsa,30), (Maria,40), and (Kevin,30), where each pair consists of the person's name and the age of the person.
Can we define a function?
If we can, what is the domain, the codomain/target, and the range of this function?



Yes, this relation defines a function *f*, where *f*(Tom) = *22*, *f*(Elsa) = *30*, *f*(Maria) = *40*, and *f*(Kevin) = *30*.

3.1 Definition of functions

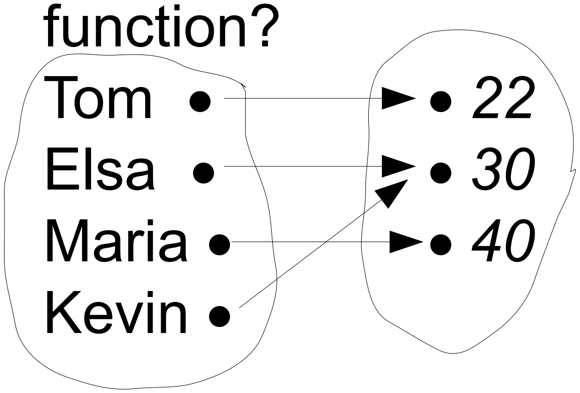
there is another way to define a function:

A **relation from A to B** is a subset of **A** $\times$ **B**. denotation: R

A relation from **A** to **B** that contains one and only one ordered pair *(a,b)* for every element $a \in$ **A**, defines a **function *f* from A to B**.

This function is defined by the assignment *f(a) = b*, where *(a,b)* is the unique ordered pair that has *a* as its first element.

Example 1:
R is the relation consisting of pairs: (Tom,22), (Elsa,30), (Maria,40), and (Kevin,30), where each pair consists of the person's name and the age of the person.
Can we define a function?
If we can, what is the domain, the codomain/target, and the range of this function?

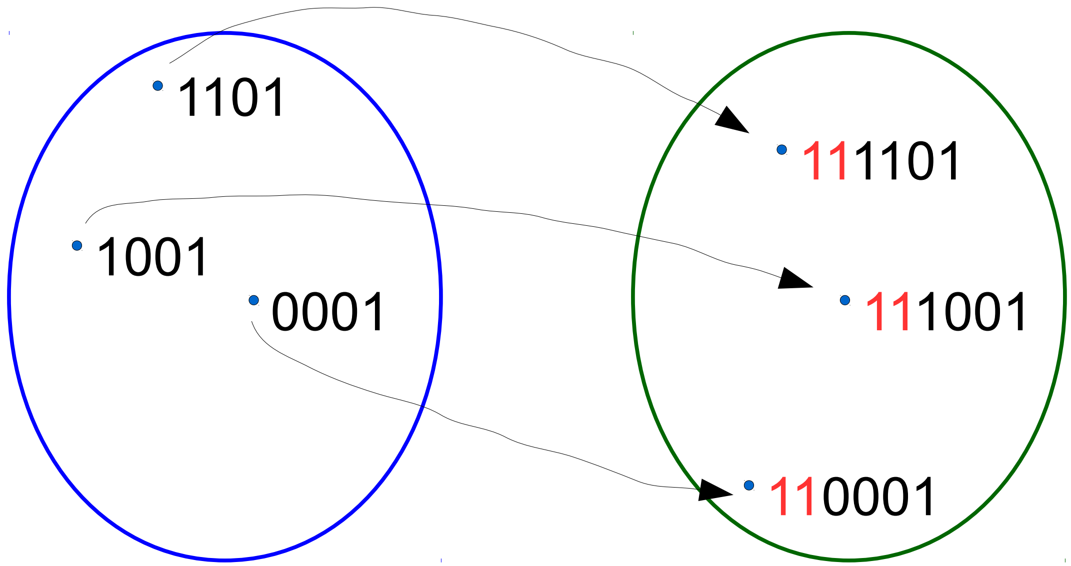


Yes, this relation defines a function *f*, where *f*(Tom) = *22*, *f*(Elsa) = *30*, *f*(Maria) = *40*, and *f*(Kevin) = *30*.
domain: {Tom, Elsa, Maria, Kevin}
Codomain/target: positive integers < *140*
range: {*22*,*30*,*40*}

3.1 Definition of functions

Example 2:

Let's define $f : \{0,1\}^4 \rightarrow \{0, 1\}^6$ such that for $x \in \{0,1\}^4$, $f(x) = 11x$



domain = all bit strings of length 4

Codomain/target = all bit strings of length 6

range = all bit strings of length 6 that start with 11

Practice Problems

1. R is the relation consisting of pairs: (Tom,22), (Elsa,30), (Maria,40), and (Kevin,30), (Tom,54) where each pair consists of the person's name and the age of the person.

Can we define a function?

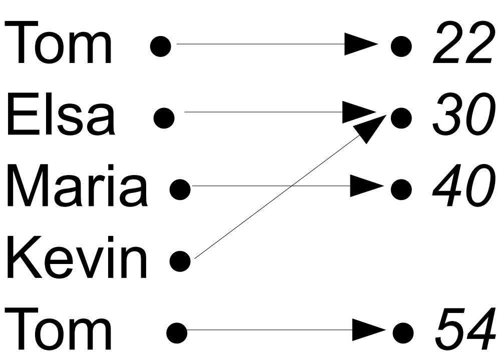
If we can, what is the domain, the codomain/target, and the range of this function?

Practice Problems

1. R is the relation consisting of pairs: (Tom,22), (Elsa,30), (Maria,40), and (Kevin,30), (Tom,54) where each pair consists of the person's name and the age of the person.

Can we define a function?

If we can, what is the domain, the codomain/target, and the range of this function?

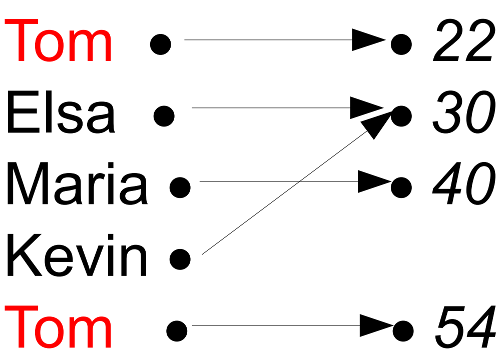


Practice Problems

1. R is the relation consisting of pairs: (Tom,22), (Elsa,30), (Maria,40), and (Kevin,30), (Tom,54) where each pair consists of the person's name and the age of the person.

Can we define a function?

If we can, what is the domain, the codomain/target, and the range of this function?

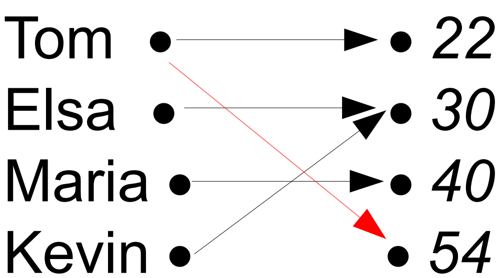


Practice Problems

1. R is the relation consisting of pairs: (Tom,22), (Elsa,30), (Maria,40), and (Kevin,30), (Tom,54) where each pair consists of the person's name and the age of the person.

Can we define a function?

If we can, what is the domain, the codomain/target, and the range of this function?



not a function

Practice Problems

2. Find the *domain* and the *range* of given functions

a) the function that assigns to a bit string the number of bits in the string

Practice Problems

2. Find the *domain* and the *range* of given functions

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

codomain/target: $\mathbf{Z}^+$

domain: the set of all bit strings

range: $\mathbf{Z}^+$ (positive integers)

Practice Problems

2. Find the *domain* and the *range* of given functions

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

domain: the set of all bit strings

codomain/target: $\mathbf{Z}^+$

range: $\mathbf{Z}^+$ (positive integers)

b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

Practice Problems

2. Find the *domain* and the *range* of given functions

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

domain: the set of all bit strings

codomain/target: $\mathbf{Z}^+$

range: $\mathbf{Z}^+$ (positive integers)

b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

f(1) = 1, f(2) = 1, f(3) = 1, f(4)=4, f(5)=4, f(6) = 4, f(7) = 4, f(8) = 4, f(9) = 9..

Practice Problems

2. Find the *domain* and the *range* of given functions

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string
codomain: $\mathbf{Z}^+$

domain: the set of all bit strings
range: $\mathbf{Z}^+$ (positive integers)

b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

f(1) = 1, f(2) = 1, f(3) = 1, f(4)=4, f(5)=4, f(6) = 4, f(7) = 4, f(8) = 4, f(9) = 9..
domain: $\mathbf{Z}^+$ *range*: {1,4,9,16,25,36,49,64,81,100,121,...}
codomain: $\mathbf{Z}^+$

Practice Problems

2. Find the *domain* and the *range* of given functions

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

codomain: $\mathbf{Z}^+$

domain: the set of all bit strings

range: $\mathbf{Z}^+$ (positive integers)

b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

$f(1) = 1$, $f(2) = 1$, $f(3) = 1$, $f(4)=4$, $f(5)=4$, $f(6) = 4$, $f(7) = 4$, $f(8) = 4$, $f(9) = 9$..

domain: $\mathbf{Z}^+$ *range*: {1,4,9,16,25,36,49,64,81,100,121,...}

codomain: $\mathbf{Z}^+$

c) $f(n) = n-1$; $f: \mathbf{Z} \to \mathbf{Z}$

Practice Problems

2. Find the *domain* and the *range* of given functions

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

domain: the set of all bit strings

codomain: $\mathbf{Z}^+$

range: $\mathbf{Z}^+$ (positive integers)

b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

f(1) = 1, f(2) = 1, f(3) = 1, f(4)=4, f(5)=4, f(6) = 4, f(7) = 4, f(8) = 4, f(9) = 9..

domain: $\mathbf{Z}^+$ *range*: {1,4,9,16,25,36,49,64,81,100,121,...}

codomain: $\mathbf{Z}^+$

c) f(n) = n-1; f: $\mathbf{Z} \to \mathbf{Z}$

domain: $\mathbf{Z}$ *codomain*: $\mathbf{Z}$ *range*: $\mathbf{Z}$

3.1 Definition of functions

Two functions are equal when they have the same domain, codomain/target and map the elements of their common domain to the same elements in their common codomain/target.

Example 1:

f: **R** → **R** $f(x) = 2x^2-4x$
g: **R** → **R** $g(x) = 2(x^2-2)$

f(x) and *g(x)* are equal.

Example 2:

f: {a,b,c} → {1,2,3}   f(a) = 1, f(b) = 2, f(c) = 3
g: {a,b,c} → {1,2,3}
g(x) = "index of the letter x in the ordered tuple (a,b,c)"

f(x) and g(x) are equal

3.2 Floor and ceiling functions

the floor function assigns to the real number *x* the largest integer that is less than or equal to *x*.

denotation: $\lfloor x \rfloor$

the ceiling function assigns to the real number *x* the smallest integer that is greater than or equal to *x*.

denotation: $\lceil x \rceil$

3.2 Floor and ceiling functions

the floor function assigns to the real number *x* the largest integer that is less than or equal to *x*.

denotation: $\lfloor x \rfloor$

the ceiling function assigns to the real number *x* the smallest integer that is greater than or equal to *x*.

denotation: $\lceil x \rceil$

Example 3: Find

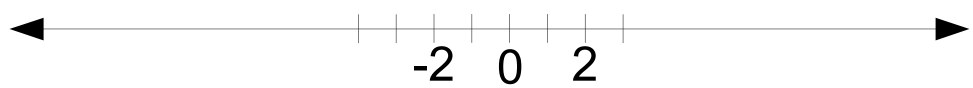
a) $\lfloor \frac{3}{4} \rfloor$

b) $\lceil 0.5 \rceil$

c) $\lceil 2 \rceil$

d) $\lfloor \frac{-3}{2} \rfloor$

e) $\lceil -2.3 \rceil$



3.2 Floor and ceiling functions

the floor function assigns to the real number *x* the largest integer that is less than or equal to *x*.

<u>denotation</u>: $\lfloor x \rfloor$

the ceiling function assigns to the real number *x* the smallest integer that is greater than or equal to *x*.

<u>denotation</u>: $\lceil x \rceil$

Example 3: Find

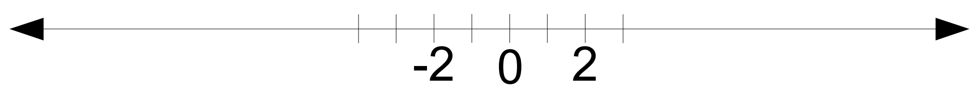
a) $\lfloor \frac{3}{4} \rfloor = \lfloor 0.75 \rfloor = 0$

b) $\lceil 0.5 \rceil$

c) $\lceil 2 \rceil$

d) $\lfloor \frac{-3}{2} \rfloor$

e) $\lceil -2.3 \rceil$



3.2 Floor and ceiling functions

the floor function assigns to the real number *x* the largest integer that is less than or equal to *x*.

denotation: $\lfloor x \rfloor$

the ceiling function assigns to the real number *x* the smallest integer that is greater than or equal to *x*.

denotation: $\lceil x \rceil$

Example 3: Find

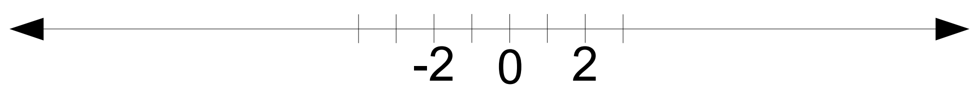
a) $\lfloor \frac{3}{4} \rfloor = \lfloor 0.75 \rfloor = 0$

b) $\lceil 0.5 \rceil = 1$

c) $\lceil 2 \rceil$

d) $\lfloor \frac{-3}{2} \rfloor$

e) $\lceil -2.3 \rceil$



3.2 Floor and ceiling functions

the floor function assigns to the real number *x* the largest integer that is less than or equal to *x*.

<u>denotation</u>: $\lfloor x \rfloor$

the ceiling function assigns to the real number *x* the smallest integer that is greater than or equal to *x*.

<u>denotation</u>: $\lceil x \rceil$

Example 3: Find

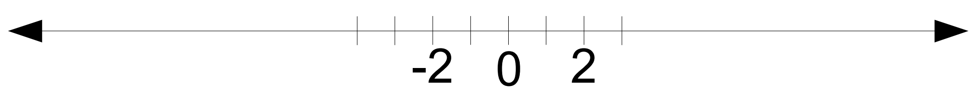
a) $\lfloor \frac{3}{4} \rfloor = \lfloor 0.75 \rfloor = 0$

b) $\lceil 0.5 \rceil = 1$

c) $\lceil 2 \rceil = \lfloor 2 \rfloor = 2$

d) $\lfloor \frac{-3}{2} \rfloor$

e) $\lceil -2.3 \rceil$



3.2 Floor and ceiling functions

the floor function assigns to the real number *x* the largest integer that is less than or equal to *x*.

denotation: $\lfloor x \rfloor$

the ceiling function assigns to the real number *x* the smallest integer that is greater than or equal to *x*.

denotation: $\lceil x \rceil$

Example 3: Find

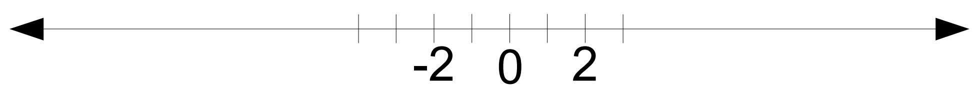
a) $\lfloor \frac{3}{4} \rfloor = \lfloor 0.75 \rfloor = 0$

b) $\lceil 0.5 \rceil = 1$

c) $\lceil 2 \rceil = \lfloor 2 \rfloor = 2$

d) $\lfloor \frac{-3}{2} \rfloor = \lfloor -1.5 \rfloor = -2$

e) $\lceil -2.3 \rceil$



3.2 Floor and ceiling functions

the floor function assigns to the real number *x* the largest integer that is less than or equal to *x*.

<u>denotation</u>: $\lfloor x \rfloor$

the ceiling function assigns to the real number *x* the smallest integer that is greater than or equal to *x*.

<u>denotation</u>: $\lceil x \rceil$

Example 3: Find

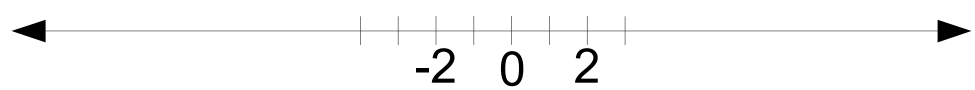
a) $\lfloor \frac{3}{4} \rfloor = \lfloor 0.75 \rfloor = 0$

b) $\lceil 0.5 \rceil = 1$

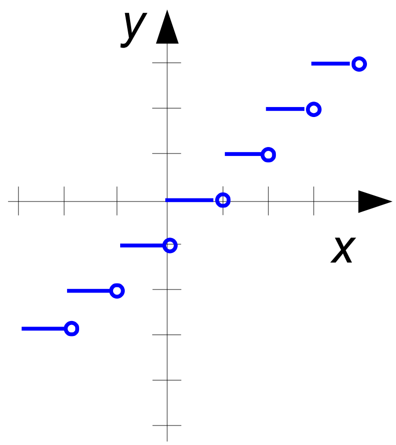
c) $\lceil 2 \rceil = \lfloor 2 \rfloor = 2$

d) $\lfloor \frac{-3}{2} \rfloor = \lfloor -1.5 \rfloor = -2$

e) $\lceil -2.3 \rceil = -2$



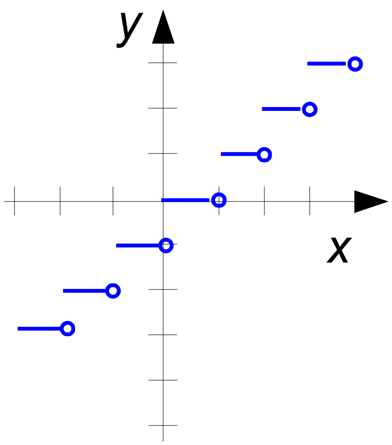
Example 4:
Let's draw the graphs of the floor and ceiling functions, $f(x) = \lfloor x \rfloor$, $g(x) = \lceil x \rceil$



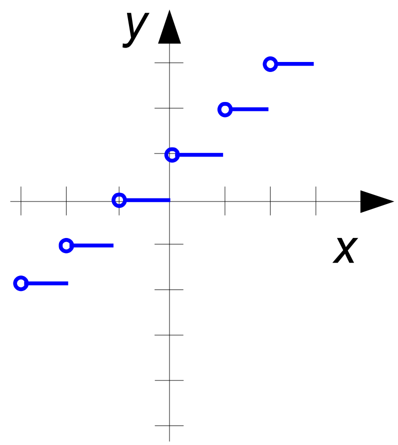
$f(x) = \lfloor x \rfloor$

3.2 Floor and ceiling functions

Example 4:
Let's draw the graphs of the floor and ceiling functions, $f(x) = \lfloor x \rfloor$, $g(x) = \lceil x \rceil$



$f(x) = \lfloor x \rfloor$



$g(x) = \lceil x \rceil$

Factorial Function

the factorial function $f : \mathbf{N} \to \mathbf{Z}^+$

$f(n) = n!$

one of the definitions: $0! = 1$ $1! = 1$ $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

Factorial Function

the factorial function f : $\mathbf{N} \rightarrow \mathbf{Z}^+$

$f(n) = n!$

one of the definitions: $0! = 1$ $1! = 1$ $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

Example 5: Find
a) 3!
b) 7!

Factorial Function

the factorial function f : $\mathbf{N} \rightarrow \mathbf{Z}^+$

f(n) = n!

one of the definitions: $0! = 1$ $1! = 1$ $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

Example 5: Find

a) $3! = 1 \cdot 2 \cdot 3$

b) $7!$

Factorial Function

the factorial function $f : \mathbf{N} \to \mathbf{Z}^+$

$f(n) = n!$

one of the definitions: $0! = 1$ $1! = 1$ $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

Example 5: Find

a) $3! = 1 \cdot 2 \cdot 3 = 6$

b) $7!$

Factorial Function

the factorial function f : $\mathbf{N} \rightarrow \mathbf{Z}^+$

f(n) = n!

one of the definitions: $0! = 1$ $1! = 1$ $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

Example 5: Find

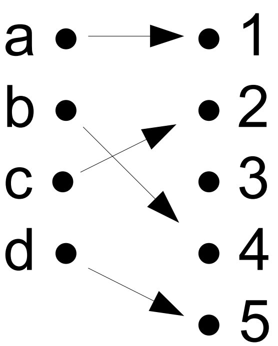
a) $3! = 1 \cdot 2 \cdot 3 = 6$

b) $7! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 = 5040$

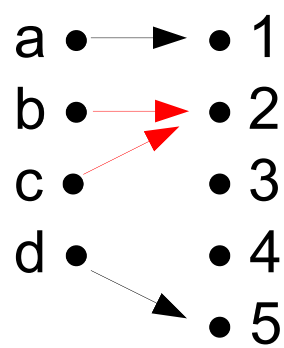
3.3 Properties of functions

Let f be a function.

If f never assigns the same value to two different domain elements, then it is called one-to-one or injective, i.e.
$f(a) \neq f(b)$ for all a and b, such that $a \neq b$; or
$f(a) = f(b)$ implies that $a=b$



one-to-one



not one-to-one

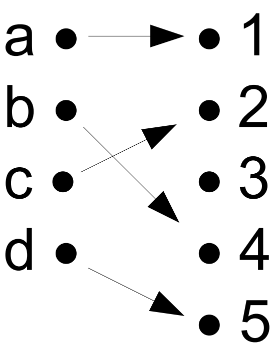
Example 1:
determine whether the given functions are one-to-one

a) $f(x) = x^3$;   f: $\mathbf{Z} \rightarrow \mathbf{Z}$

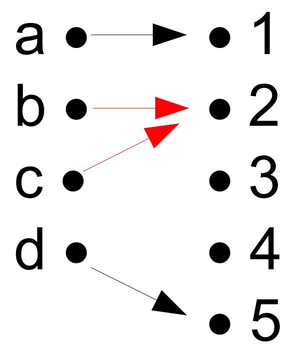
b) f: $\{1,2,3,4\} \rightarrow \{6,7,8\}$, with $f(1) = 6$, $f(2) = 7$, $f(3) = 8$, and $f(4) = 7$

Let *f* be a function.

If *f* never assigns the same value to two different domain elements, then it is called one-to-one or injective, i.e.
$f(a) \neq f(b)$ for all a and b, such that $a \neq b$; *or*
$f(a) = f(b)$ implies that $a=b$



one-to-one



not one-to-one

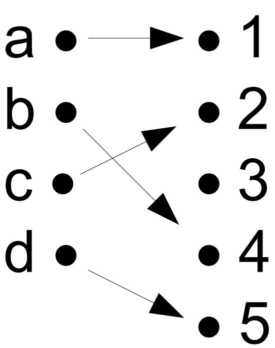
Example 1:
determine whether the given functions are one-to-one

a) $f(x) = x^3$;   f: $\mathbf{Z} \rightarrow \mathbf{Z}$
  one-to-one (power 3 preserves the sign)
b) *f*: $\{1,2,3,4\} \rightarrow \{6,7,8\}$, with $f(1) = 6$, $f(2) = 7$, $f(3) = 8$, and $f(4) = 7$

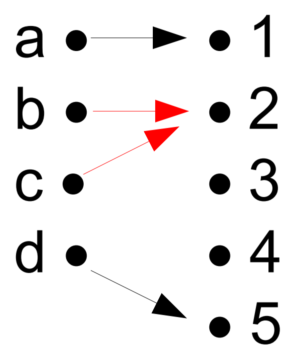
3.3 Properties of functions

Let *f* be a function.

If *f* never assigns the same value to two different domain elements, then it is called one-to-one or injective, i.e.
$f(a) \neq f(b)$ for all *a* and *b*, such that $a \neq b$; *or*
$f(a) = f(b)$ implies that $a=b$

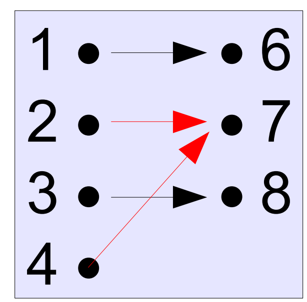


one-to-one



not one-to-one

Example 1:
determine whether the given functions are one-to-one



a) $f(x) = x^3$; f: $\mathbf{Z} \rightarrow \mathbf{Z}$
  one-to-one (power 3 preserves the sign)

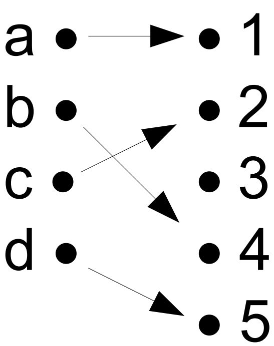
b) *f*: $\{1,2,3,4\} \rightarrow \{6,7,8\}$, with $f(1) = 6$, $f(2) = 7$, $f(3) = 8$, and $f(4) = 7$

3.3 Properties of functions

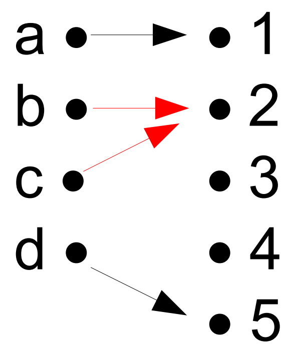
Let *f* be a function.

If *f* never assigns the same value to two different domain elements, then it is called one-to-one or injective, i.e.
$f(a) \neq f(b)$ for all a and b, such that $a \neq b$; *or*

$f(a) = f(b)$ implies that $a=b$

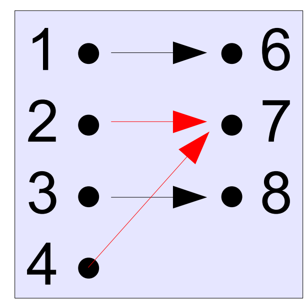


one-to-one



not one-to-one

Example 1:
determine whether the given functions are one-to-one



a) $f(x) = x^3$; f: $\mathbf{Z} \rightarrow \mathbf{Z}$
one-to-one (power 3 preserves the sign)

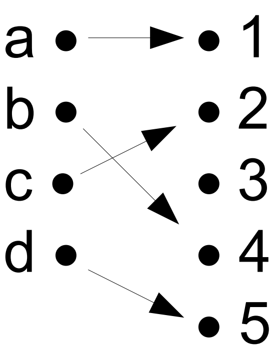
b) *f*: $\{1,2,3,4\} \rightarrow \{6,7,8\}$, with $f(1) = 6$, $f(2) = 7$, $f(3) = 8$, and $f(4) = 7$
not one-to-one, because of $f(2) = 7$ and $f(4) = 7$

3.3 Properties of functions

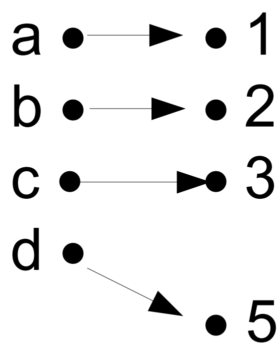
Let f be a function, f: **A** $\rightarrow$ **B**

If for every element $b \in$ **B** there is an element $a \in$ **A**, such that $f(a) = b$, then function f is onto or surjective, i.e. range is equal to the codomain/target

$\forall y \exists x\ (f(x)=y)$



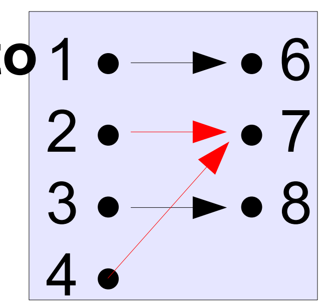
one-to-one, but not onto



one-to-one, onto

Example 2 (same functions as in example 4):

determine whether the given functions are **one-to-one** and **onto**



a) $f(x) = x^3$; $\quad$ f: **Z** $\rightarrow$ **Z**

one-to-one (power 3 preserves the sign)

not onto, because we cannot find such an x that $x^3 = 2$

b) f: $\{1,2,3,4\} \rightarrow \{6,7,8\}$, with $f(1) = 6$, $f(2) = 7$, $f(3) = 8$, and $f(4) = 7$

not one-to-one, because of $f(2) = 7$ and $f(4) = 7$

onto

3.3 Properties of functions

Function f is one-to-one correspondence or bijective if it is both one-to-one and onto.

Example 3:
Let f:{$1,2,3,4,5$} $\rightarrow$ {$6,7,8,9,10$}, with $f(1) = 10$, $f(2) = 9$, $f(3) = 8$, $f(4) = 7$, $f(5) = 6$. Is f bijective?

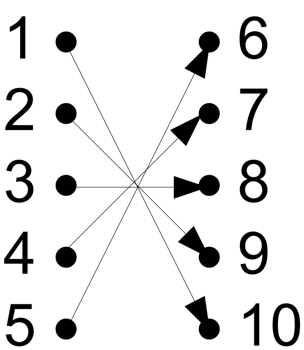
Solution:

3.3 Properties of functions

Function *f* is one-to-one correspondence or bijective if it is both one-to-one and onto.

Example 3:
Let f:{$1,2,3,4,5$} $\rightarrow$ {$6,7,8,9,10$}, with $f(1) = 10$, $f(2) = 9$, $f(3) = 8$, $f(4) = 7$, $f(5) = 6$. Is f bijective?

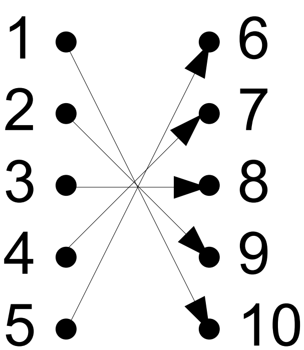
<ins>Solution</ins>:



Function *f* is one-to-one correspondence or bijective if it is both one-to-one and onto.

Example 3:
Let f:{$1,2,3,4,5$} $\rightarrow$ {$6,7,8,9,10$}, with $f(1) = 10$, $f(2) = 9$, $f(3) = 8$, $f(4) = 7$, $f(5) = 6$. Is f bijective?

<u>Solution</u>:



We can see that for every element from the codomain {6,7,8,9,10} there is an element from the domain {1,2,3,4,5}, therefore it is onto.

We also can see that no two different elements from the domain have the same image, therefore it is one-to-one.

The given function is bijective.

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

a) the function that assigns to a bit string the number of bits in the string

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

(one-to-one) Can two different bit strings have the same “length”?

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

(one-to-one) Can two different bit strings have the same “length”? Yes therefore it is not one-to-one

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

(one-to-one) Can two different bit strings have the same "length"? Yes therefore it is not one-to-one

(onto) for any value *n* from the range (positive integer) can we find a bit string that has *n* bits?

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

(one-to-one) Can two different bit strings have the same "length"? Yes therefore it is not one-to-one

(onto) for any value *n* from the range (positive integer) can we find a bit string that has *n* bits? Yes
therefore it is onto

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

(one-to-one) Can two different bit strings have the same “length”? Yes therefore it is not one-to-one

(onto) for any value *n* from the range (positive integer) can we find a bit string that has *n* bits? Yes
therefore it is onto

b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

(one-to-one) Can two different bit strings have the same "length"? Yes therefore it is not one-to-one

(onto) for any value *n* from the range (positive integer) can we find a bit string that has *n* bits? Yes
therefore it is onto

b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

$f(1) = 1$, $f(2) = 1$, $f(3) = 1$, $f(4)=4$, $f(5)=4$, $f(6) = 4$, $f(7) = 4$, $f(8) = 4$, $f(9) = 9$..

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

(one-to-one) Can two different bit strings have the same "length"? Yes therefore it is not one-to-one

(onto) for any value *n* from the range (positive integer) can we find a bit string that has *n* bits? Yes
therefore it is onto

b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

$f(1) = 1, f(2) = 1, f(3) = 1, f(4)=4, f(5)=4, f(6) = 4, f(7) = 4, f(8) = 4, f(9) = 9..$

not one-to-one, because of $f(4)=4$, $f(5)=4$, for example

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

a) the function that assigns to a bit string the number of bits in the string

f(bit string) = # bits in the string

(one-to-one) Can two different bit strings have the same "length"? Yes therefore it is not one-to-one

(onto) for any value *n* from the range (positive integer) can we find a bit string that has *n* bits? Yes
therefore it is onto

b) the function that assigns to each positive integer the largest perfect square not exceeding this integer

$f(1) = 1, f(2) = 1, f(3) = 1, f(4)=4, f(5)=4, f(6) = 4, f(7) = 4, f(8) = 4, f(9) = 9..$

not one-to-one, because of $f(4)=4$, $f(5)=4$, for example
not onto, because 2 is from the *codomain/target*, but not from the *range* of *f*

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

c) $f(n) = n-1$; f: $\mathbf{Z} \to \mathbf{Z}$

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

c) $f(n) = n-1$;   f: **Z** → **Z**

f(-2) *= -2-1=**-3**,* ***f(-1)*** *= -1-1=**-2**,* ***f(0)*** *= 0-1=**-1**,* ***f(1)*** *= 1-1=**0**,* ***f(2)*** *= 2-1=**1***

one-to-one, because for two different values from the domain there are two different images;

Practice Problems

Determine whether they are *one-to-one* (*injective*), *onto* (*surjective*) or both (i.e. *bijective*).

c) f(n) = n-1;   f: **Z** $\rightarrow$ **Z**

f(-2) *= -2-1=**-3**, **f(-1)** = -1-1=**-2**, **f(0)** = 0-1=**-1**, **f(1)** = 1-1=**0**, **f(2)** = 2-1=**1***

one-to-one, because for two different values from the domain there are two different images;
onto, because for any value *s* from the codomain/target, there is a value in the domain: *(s+1)*   *f(s+1) = s*
therefore, the given function is bijective