Chapter 2. Basic Structures: Sets, Functions, Sequences, Sums

2.3 Union and Intersection
2.4 More set operations
2.7 Partitions

Set operations

Let **A**, **B** be any two sets.

The union of two sets **A** and **B** is the set that contains those elements that are either in **A** or in **B**, or in both.

denotation: **A** U **B** $\quad$ **A** U **B** = { x | x ∈ **A** ∨ x ∈ **B** }

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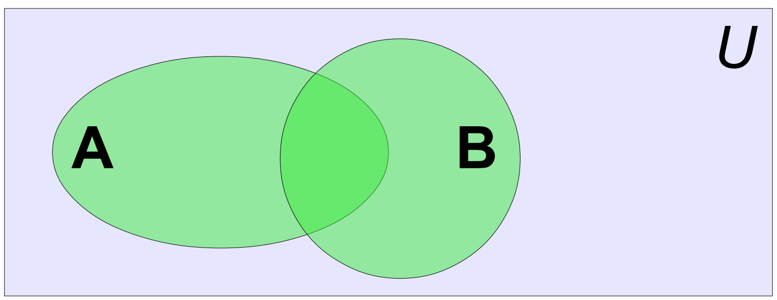

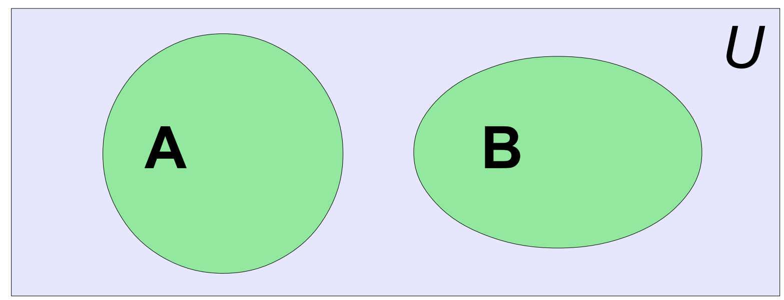

Shaded (with green) areas represent the union of sets **A** and **B**.

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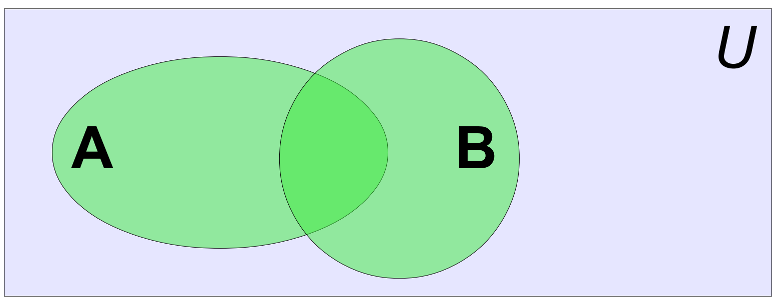

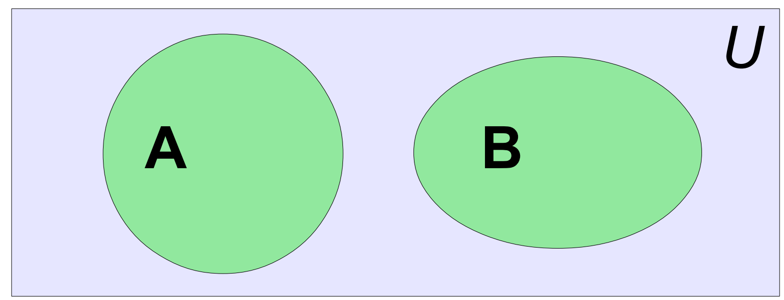

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Example 1:

Let **A**={1,2,3} and **B**={a,b,c}, then **A U B =**

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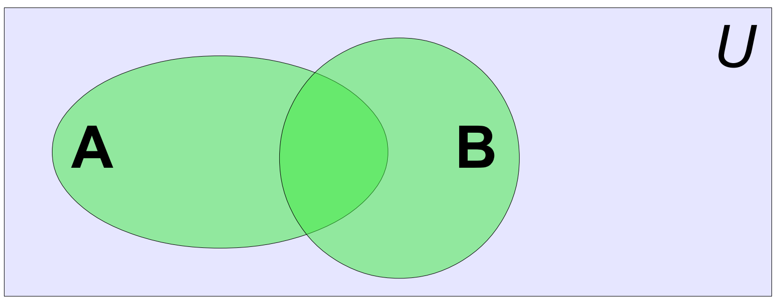

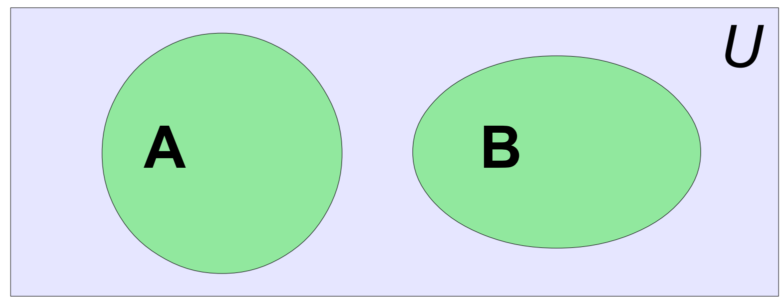

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Example 1:

Let **A**={1,2,3} and **B**={a,b,c}, then **A** U **B** = {1,2,3,a,b,c}

Set operations: intersection

Let **A**, **B** be any two sets.

The intersection of two sets **A** and **B** is the set that contains those elements that are in both **A** and **B**.

denotation: **A** ∩ **B** $\mathbf{A} \cap \mathbf{B} = \{ x \mid x \in \mathbf{A} \wedge x \in \mathbf{B} \}$

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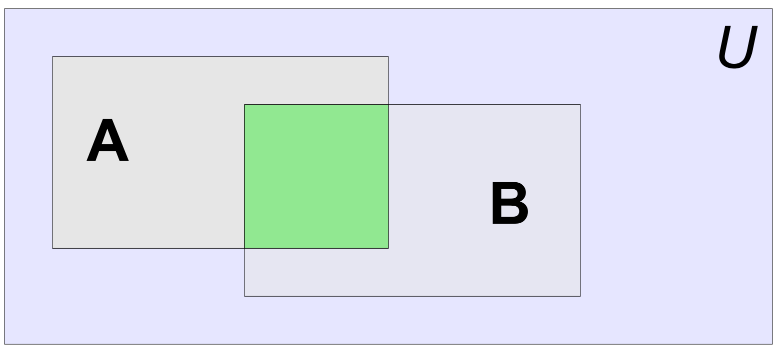

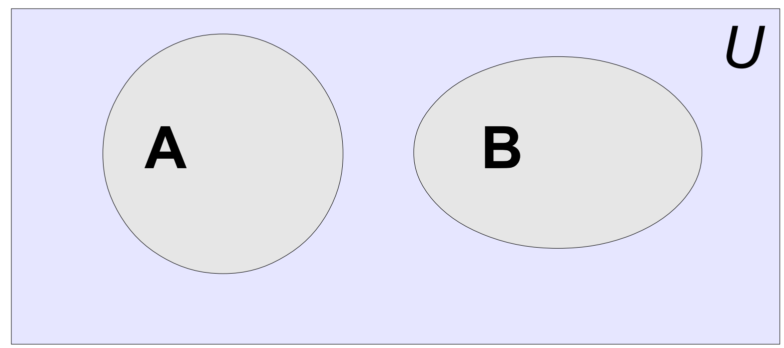

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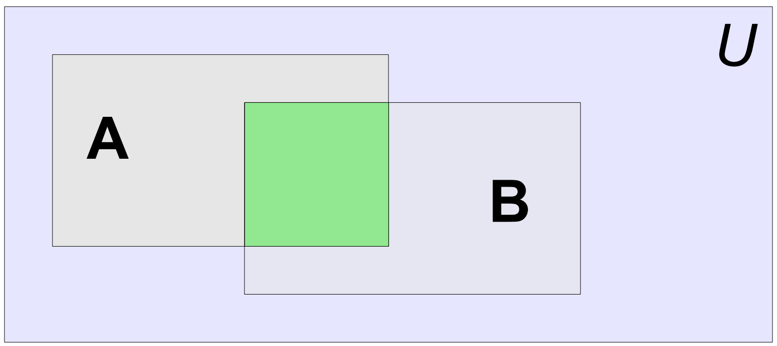

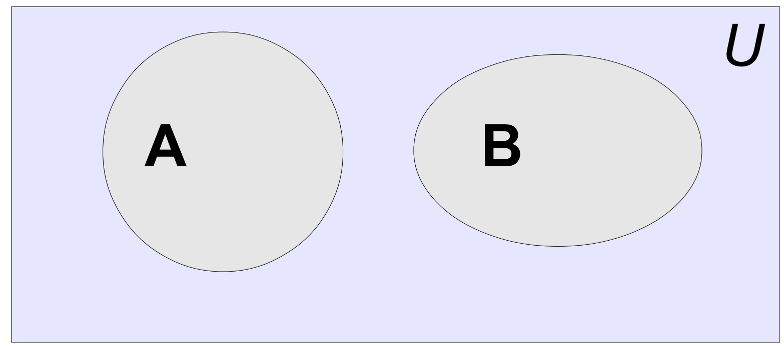

Shaded areas represent the intersection of sets **A** and **B**.

Example 2:

Let **A**={1,2,3,a,b,c} and **B**={a,b,d,1}, then $\mathbf{A} \cap \mathbf{B}$ =

Set operations: intersection

Let **A**, **B** be any two sets.

The intersection of two sets **A** and **B** is the set that contains those elements that are in both **A** and **B**.

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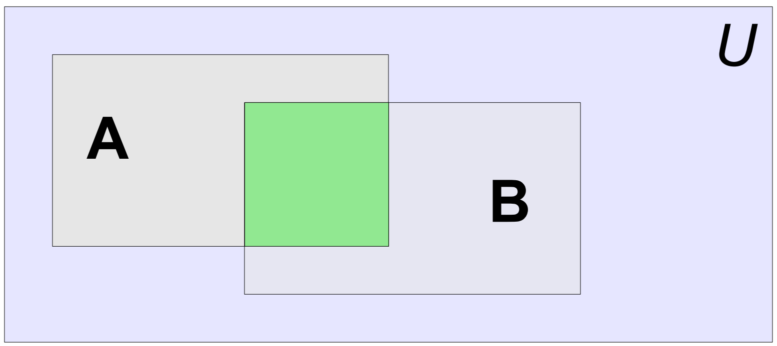
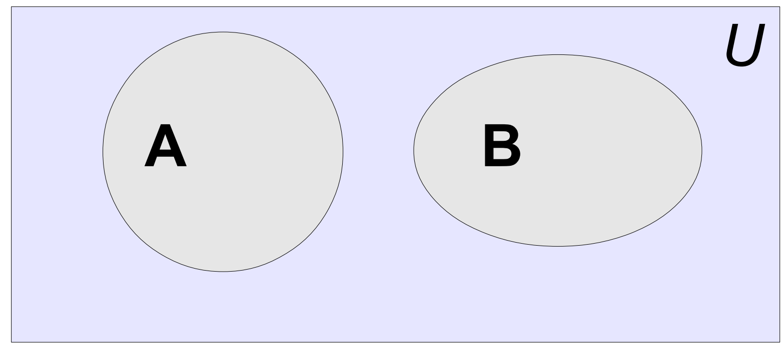




Shaded areas represent the intersection of sets **A** and **B**.

Example 2:

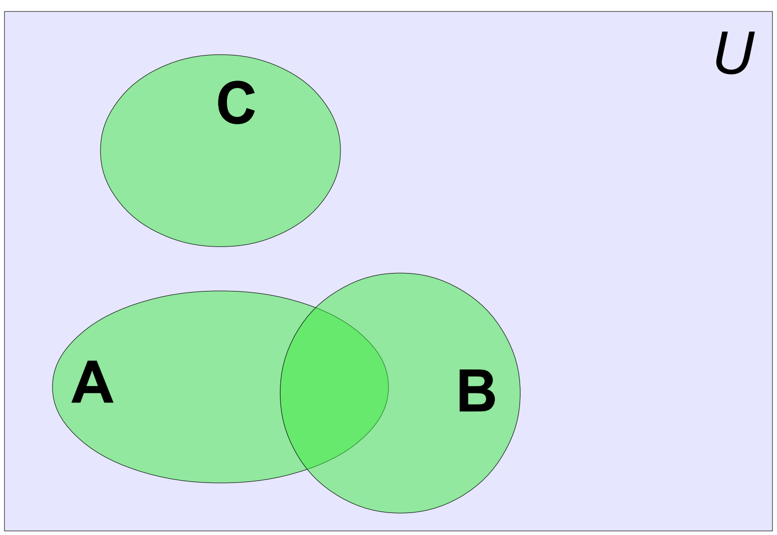
Let **A**={1,2,3,a,b,c} and **B**={a,b,d,1}, then $\mathbf{A} \cap \mathbf{B} = \{1,a,b\}$

Set operations: union

Let **A**, **B**, and **C** be any three sets.

The union of three sets **A**, **B**, and **C** is the set that contains those elements that are members of at least one of the sets.

denotation: **A** U **B** U **C** **A** U **B** U **C** = { x | x ∈ **A** ∨ x ∈ **B** ∨ x ∈ **C** }



A U **B** U **C**

Set operations: union and intersection

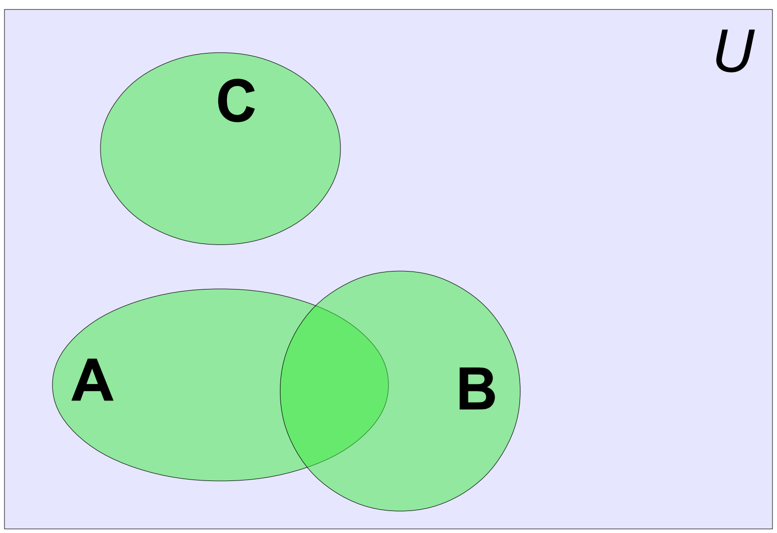
Let **A**, **B**, and **C** be any three sets.

The union of three sets **A**, **B**, and **C** is the set that contains those elements that are members of at least one of the sets.

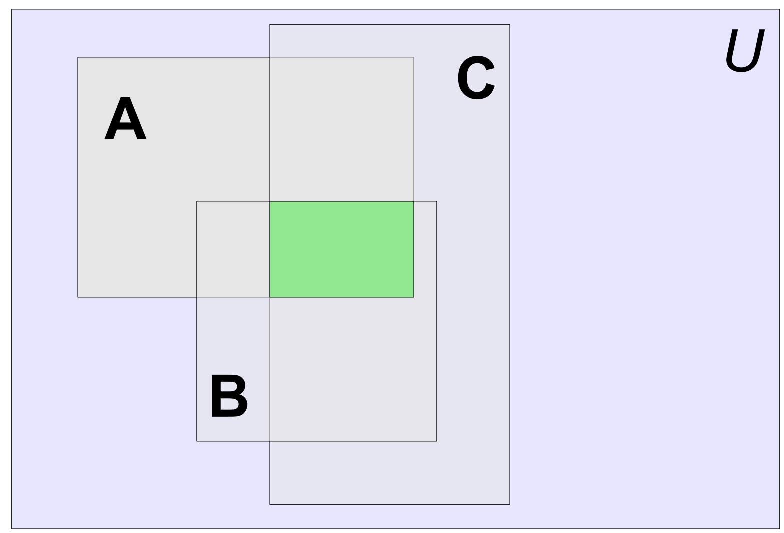
denotation: $\mathbf{A} \cup \mathbf{B} \cup \mathbf{C}$ $\mathbf{A} \cup \mathbf{B} \cup \mathbf{C} = \{ x \mid x \in \mathbf{A} \vee x \in \mathbf{B} \vee x \in \mathbf{C} \}$

The intersection of three sets **A**, **B**, and **C** is the set that contains those elements that are in each set of the collection.

denotation: $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$ $\mathbf{A} \cap \mathbf{B} \cap \mathbf{C} = \{ x \mid x \in \mathbf{A} \wedge x \in \mathbf{B} \wedge x \in \mathbf{C} \}$



$\mathbf{A} \cup \mathbf{B} \cup \mathbf{C}$



$\mathbf{A} \cap \mathbf{B} \cap \mathbf{C}$

2.4 Difference of two sets

Let **A**, **B** be any two sets.

The difference of **A** and **B** is the set containing those elements that are in **A**, but not in **B**.

denotation: **A - B** $\quad$ $\mathbf{A} - \mathbf{B} = \{ x \mid x \in \mathbf{A} \wedge x \notin \mathbf{B} \}$

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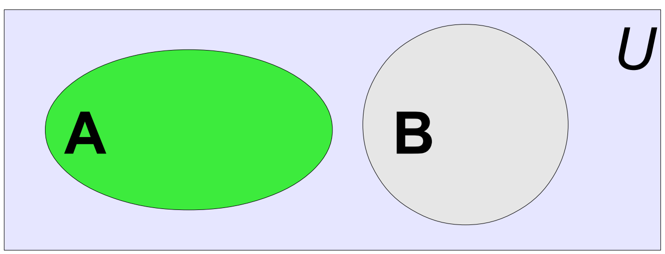
A – B is shaded



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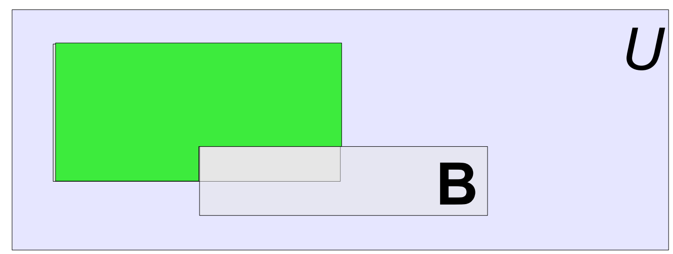

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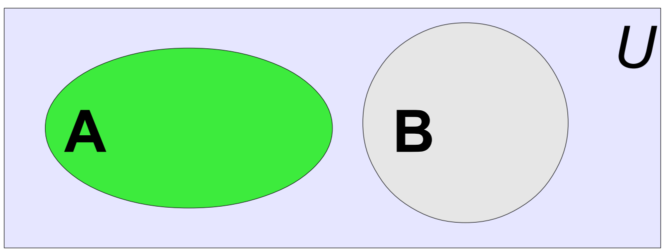

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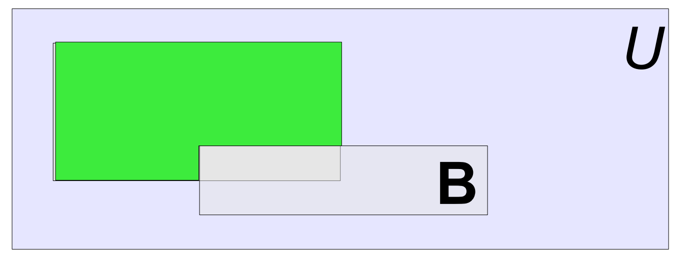

Example 3:

Let **A**={1,2,3,4,a,n,f} and **B**={5,6,7,b}, then **A ∩ B =**

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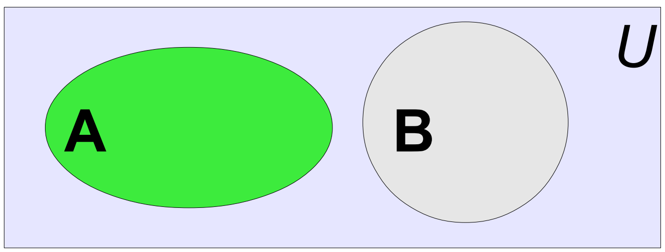

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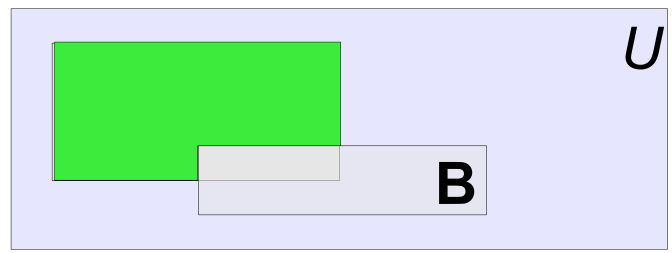

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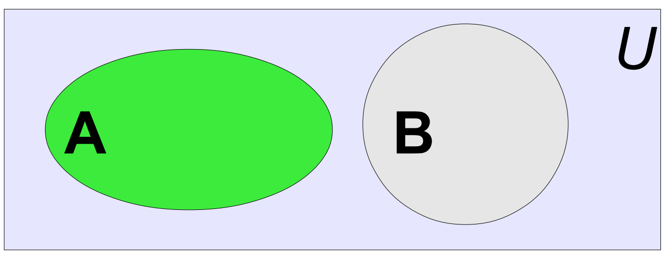

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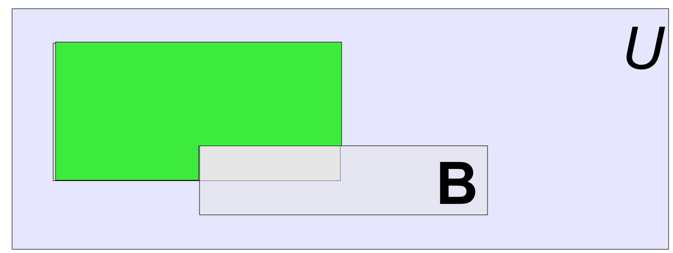

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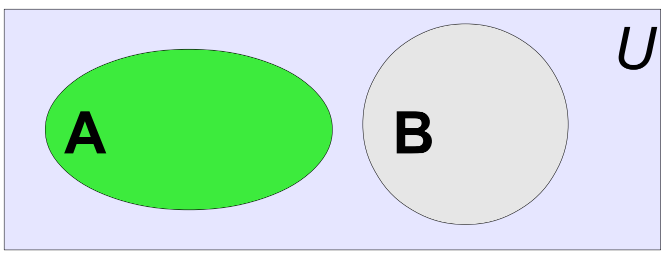

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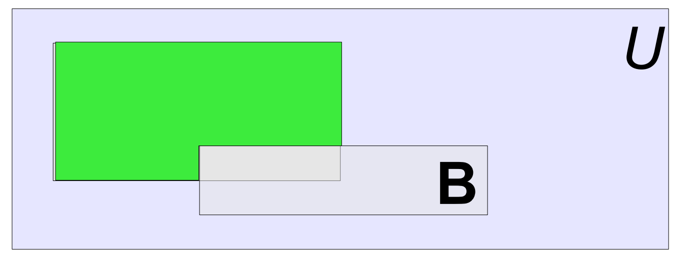


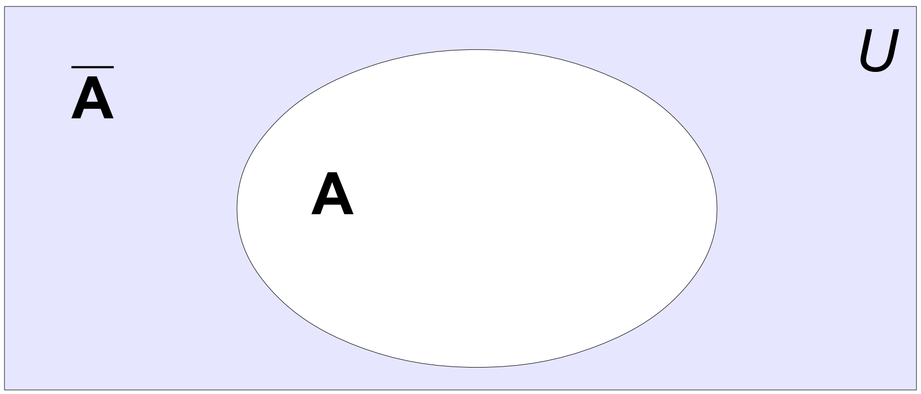
Example 4:

Let **A**={1,2,3,4,a,n,f} and **B**={1,2,3,a,b}, then **A – B** = {4,n,f}

2.4 Complement of a set

Let's assume that the universe set *U* has been specified. And let **A** be any set. Then, the compliment of the set **A** is the set of all elements of the universe set *U* that are not the elements of set A.

denotation: $\overline{\mathbf{A}}$ $\overline{\mathbf{A}} = \{ x \mid x \notin A \}$



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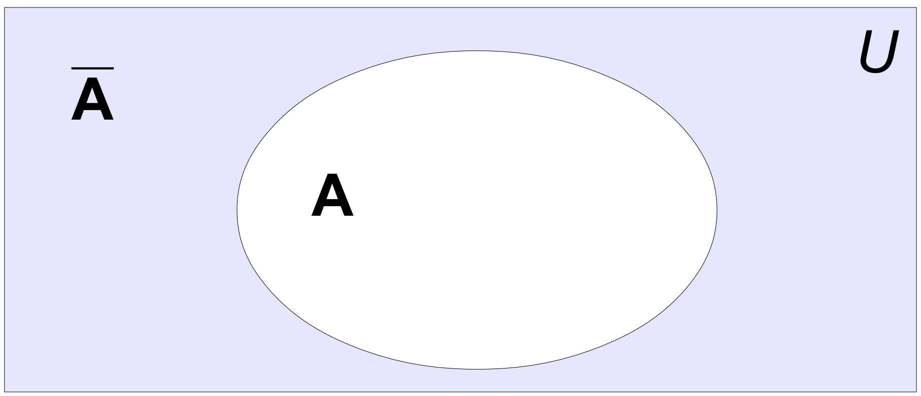

Example 5:

Let *U* be the set of all integers, and let **A**={0,1,2,3,4,...}, then

$\overline{\mathbf{A}}$ =

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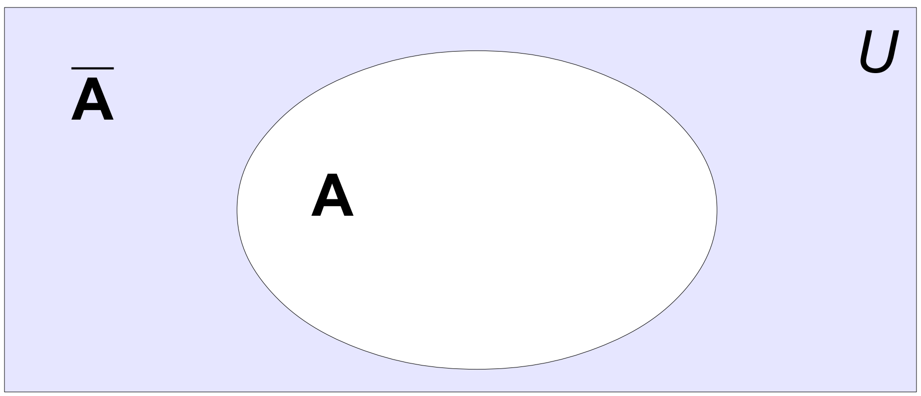
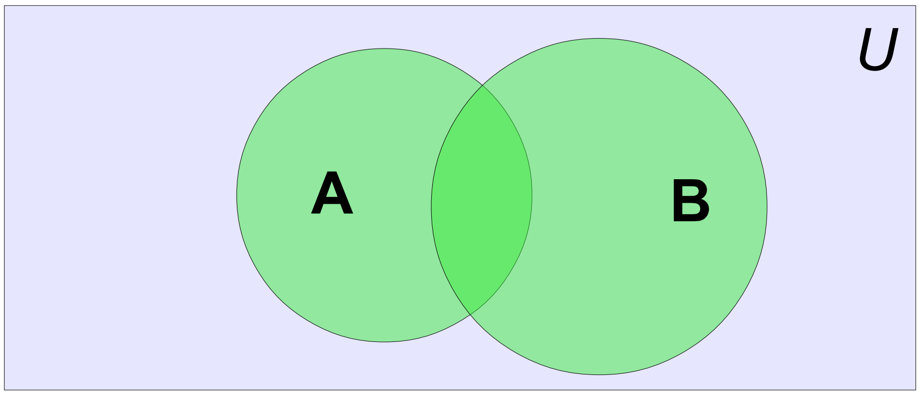


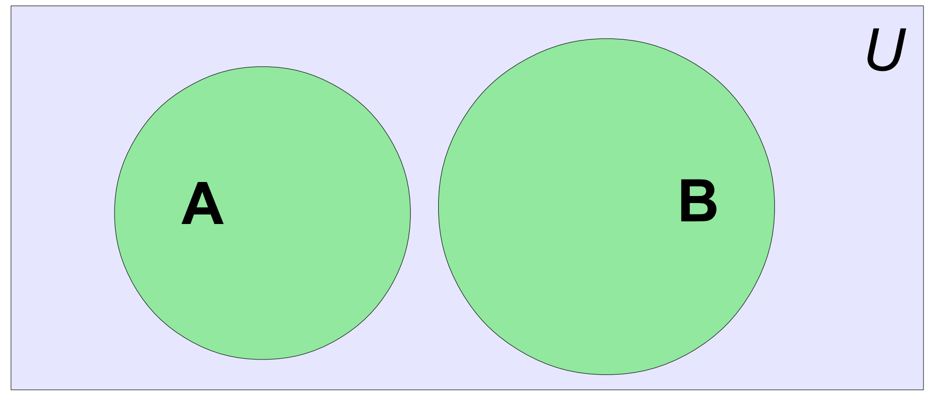
Example 5:

Let *U* be the set of all integers, and let **A**={0,1,2,3,4,...}, then
$\overline{\mathbf{A}} = \{...,-4,-3,-2,-1\}$

Theorem

$|\mathbf{A} \cup \mathbf{B}| = |\mathbf{A}| + |\mathbf{B}| - |\mathbf{A} \cap \mathbf{B}|$





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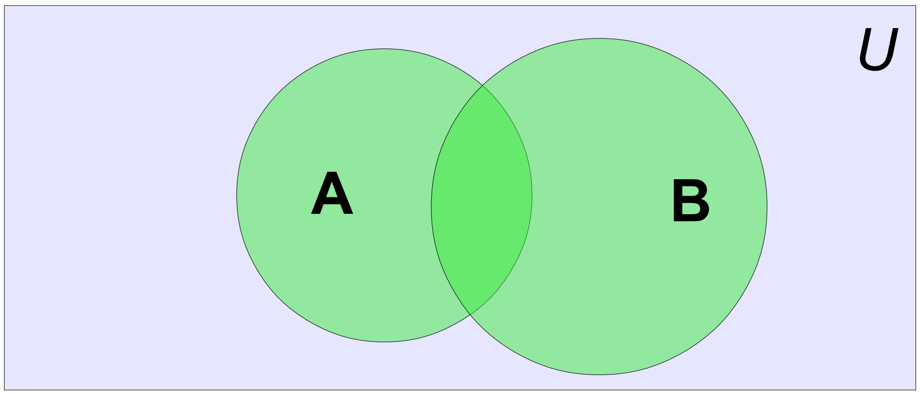
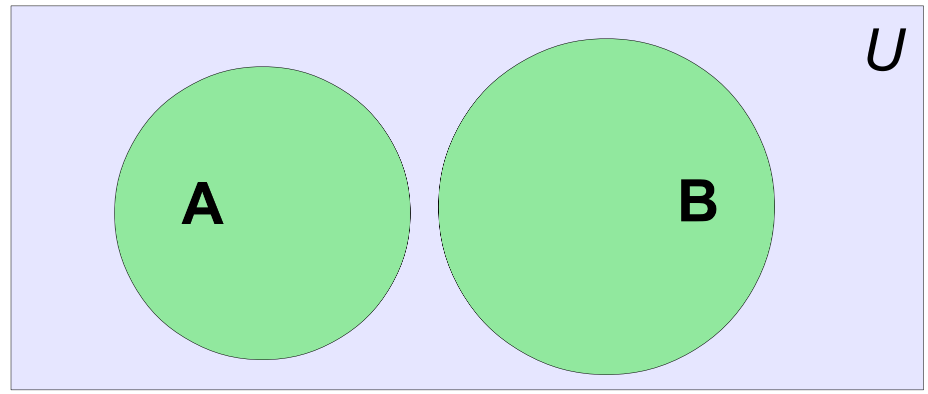




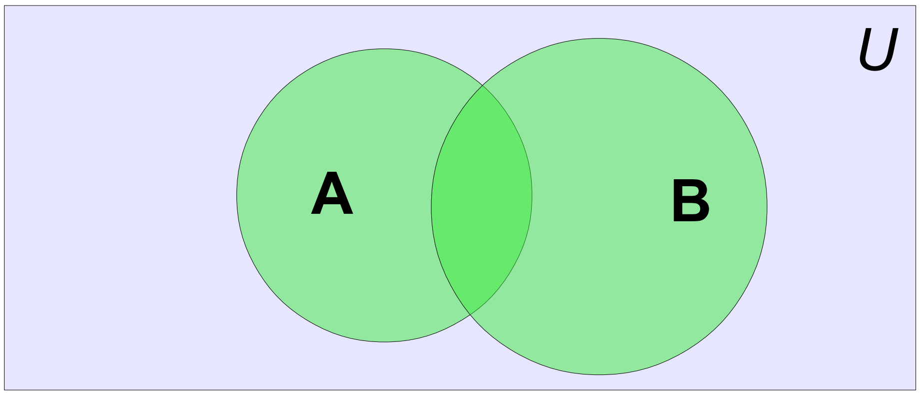
Example 6:

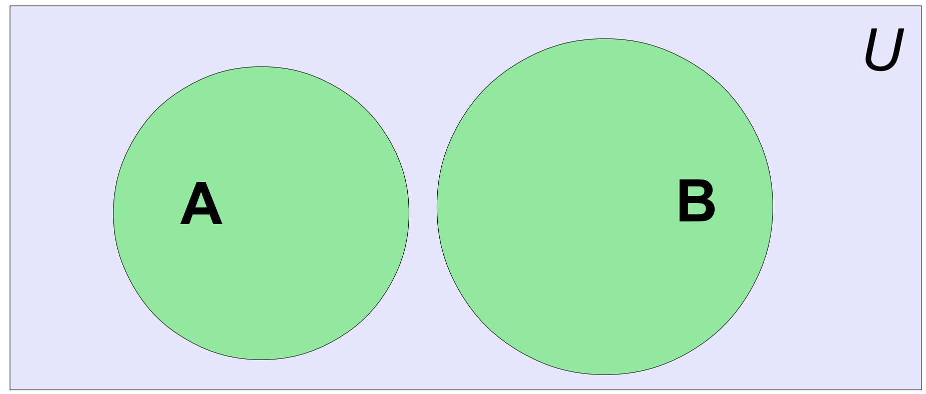
Let *U* be the set of all integers, let **A**={0,1,2,3,4}, and **B**={0,1,2,a,b,c,d,e}.

Then **A** U **B** = ? , and A ∩ B= ?

Theorem

|**A** ∪ **B**| = |**A**| + |**B**| - |**A** ∩ **B**|



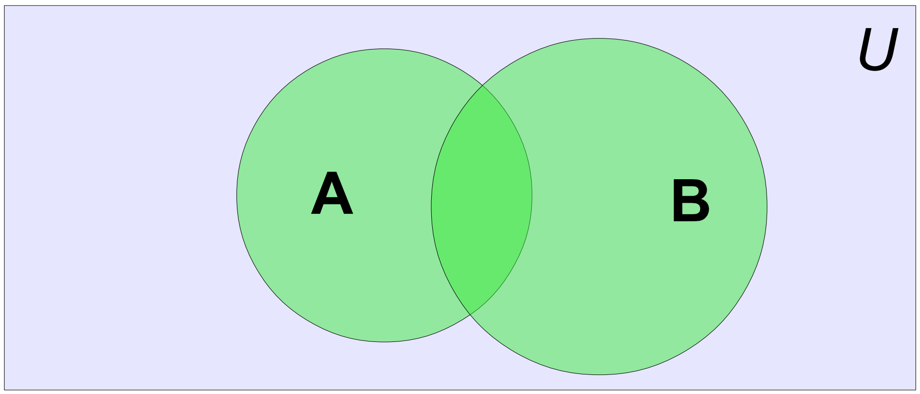


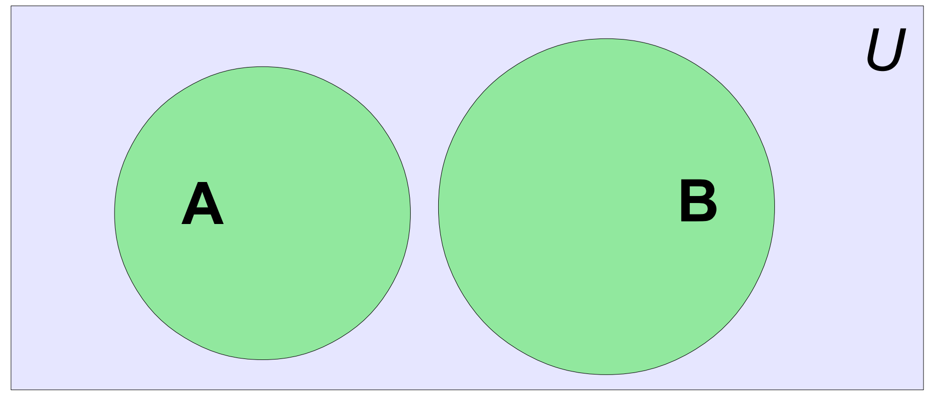
Example 6:

Let *U* be the set of all integers, let **A**={0,1,2,3,4}, and **B**={0,1,2,a,b,c,d,e}.
Then **A** ∪ **B** = {0,1,2,3,4,a,b,c,d,e}, and A ∩ B={0,1,2}

Theorem

$|\mathbf{A} \cup \mathbf{B}| = |\mathbf{A}| + |\mathbf{B}| - |\mathbf{A} \cap \mathbf{B}|$





Example 6:

Let *U* be the set of all integers, let **A**={0,1,2,3,4}, and **B**={0,1,2,a,b,c,d,e}.
Then **A** U **B** = {0,1,2,3,4,a,b,c,d,e}, and A ∩ B={0,1,2}

|A| = 5, |B| = 8, |A U B| = 10, |A ∩ B|= 3
according to the theorem the following equality should hold: 10 = 5+8-3
Indeed it holds.

Example 7:

Let **A**={1,2,3,4,5}, **B**={0,2,3,6}, and **C**={1,5}.
Find the given sets and their cardinalities.

a) **A** ∪ **B** =

b) **A** ∪ **C** =

c) **A** ∪ **B** ∪ **C** =

d) **A** ∩ **B** =

e) **A** ∩ **B** ∩ **C** =

Example 7:

Let **A**={1,2,3,4,5}, **B**={0,2,3,6}, and **C**={1,5}.
Find the given sets and their cardinalities.

a) **A** U **B** = {0,1,2,3,4,5,6}
|A U B| = 7

b) **A** U **C** =

c) **A** U **B** U **C** =

d) **A** ∩ **B** =

e) **A** ∩ **B** ∩ **C** =

Example 7:

Let **A**={1,2,3,4,5}, **B**={0,2,3,6}, and **C**={1,5}.
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a) **A** U **B** = {0,1,2,3,4,5,6}
|A U B| = 7

b) **A** U **C** = {1,2,3,4,5}
|A U C|=5

c) **A** U **B** U **C** =

d) **A** ∩ **B** =

e) **A** ∩ **B** ∩ **C** =

Example 7:

Let **A**={1,2,3,4,5}, **B**={0,2,3,6}, and **C**={1,5}.
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a) **A** U **B** = {0,1,2,3,4,5,6}
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   |**A** U **B** U **C**|=7

d) **A** ∩ **B** = {2,3}
   |**A** ∩ **B**|=2

e) **A** ∩ **B** ∩ **C** =

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Find the given sets and their cardinalities.

a) **A** U **B** = {0,1,2,3,4,5,6}
|A U B| = 7

b) **A** U **C** = {1,2,3,4,5}
|A U C|=5

c) **A** U **B** U **C** ={0,1,2,3,4,5,6}
|**A** U **B** U **C**|=7

d) **A** ∩ **B** = {2,3}
|**A** ∩ **B**|=2

e) **A** ∩ **B** ∩ **C** = ∅
|**A** ∩ **B** ∩ **C**|=0

2.7 Partitions: Disjoint sets

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The sets **A**, **B**, and **C** are pairwise disjoint, because
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The sets **A, B, C**, and **D** are not pairwise disjoint because $\mathbf{A} \cap \mathbf{D} = \{1\}$.