Chapter 1. The Foundations: Logic and Proofs

1.6 Predicates and quantifiers
1.7 Quantified statements
1.8 De Morgan's law for quantified statements

Can we express the following statement in **propositional logic**?

"Every computer has a CPU"

Can we express the following statement in **propositional logic**?

"Every computer has a CPU"

No

1.6 Predicates and Quantifiers

Let's see the new type of logic: Predicate Logic

consider the statement: "x is greater than 3" - it has two parts:

variable x (subject of a statement)

predicate (refers to a property the subject of a statement can have)

denotation: *P(x)*: "*x* is greater than 3"
- this kind of statement is neither true nor false when the value of variable is not specified.

P(x) reads as propositional function P at *x*

Once *x* is assigned a value, *P(x)* becomes a proposition that has a truth value.

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P(10,11) : *"10 = 11-17"* if we simplify the equation: *"10 = -6"* False
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1.6 Predicates and Quantifiers

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In general, a statement of the form $P(x_1, x_2, ..., x_n)$ is the value of the propositional function *P* at the *n-tuple*.

P is also called n-place predicate or n-ary predicate.

1.6 Predicates and Quantifiers

How to express words “all”, “any”, “some”, ... ?
- quantification

“all”, “any”: $\forall$ - universal quantification
“some”, “an”: $\exists$ - existential quantification

$\forall xP(x)$ is “for all values of x from the domain, such that P(x) holds”
“for any element x in the domain P(x) holds (or true)”

$\exists xP(x)$ is “there exists value of x, such that P(x) holds (or true)”

Recall the question I asked in the beginning:

Can we express the following statement in **propositional logic**?

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1.6 Predicates and Quantifiers

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Example 3:
Let $Q(x)$ be “$2 \cdot x \geq x$”. Is $\forall x Q(x)$ true? (domain: all real numbers)

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Reasoning:
$\forall x Q(x)$ stands for *"for all x, $2 \cdot x \geq x$".* In order to show that it is not true (to *disprove it*) all we need to do is to find an least one value of x, for which $2 \cdot x \geq x$ is not true, *i.e.* $2 \cdot x < x$ – *give a counterexample*
Can you think of any?

"all", "any": ∀ - universal quantification

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Can you think of any?
Think of negative numbers ...

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Can you think of any?
Think of negative numbers ...

counterexample

For example $x=-2$, *then* $Q(-2)$ is "$2\cdot(-2) \geq (-2)$", i.e. "$-4 \geq -2$" False.
Therefore $\forall xQ(x)$ is false.

1.6 Predicates and Quantifiers

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Example 4:
Let $Q(x)$ be "$2+x \geq x$". Is $\forall x Q(x)$ true? (domain: all real numbers)

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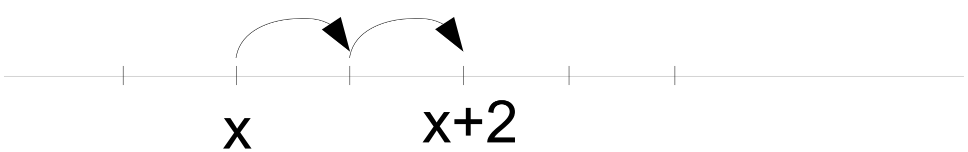
Example 4:
Let $Q(x)$ be "$2+x \geq x$". Is $\forall x Q(x)$ true? (domain: all real numbers)

<u>Reasoning</u>:
$\forall x Q(x)$ stands for *"for all x, $2+x \geq x$".* It seems to be true, because when we take any real number, and add positive number to it, it becomes greater.
May be it is more visible with a number line:



$x+2 \geq x$ True

1.6 Predicates and Quantifiers

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Example 5:
What's the truth value of $\forall xP(x)$, where P(x) is the statement "$x^2 \leq 16$" and the domain consists of the positive integers not exceeding 4?

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What do we need to check (what values)?

1.6 Predicates and Quantifiers

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What's the domain? 1, 2, 3, 4

What do we need to check (what values)? *P(1), P(2), P(3), P(4)* “all”, “any”: $\forall$ - universal quantification

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What's the domain? 1, 2, 3, 4

What do we need to check (what values)? *P(1), P(2), P(3), P(4)*
P(1): “$1^2 \leq 16$”, *P(2)*: “$2^2 \leq 16$”, *P(3)*: “$3^2 \leq 16$”, *P(4)*: “$4^2 \leq 16$”
true true true true

Therefore $P(1) \wedge P(2) \wedge P(3) \wedge P(4)$ is true, hence $\forall x P(x)$ is true.

1.6 Predicates and Quantifiers

"some", "an": $\exists$ - existential quantification

$\exists x P(x)$ is "there exists value of x, such that P(x) holds (or true)"

$\exists x P(x)$ is <u>true</u>, if *P(x)* is true for at least one *x*
$\exists x P(x)$ is <u>false</u>, if *P(x)* is false for all *x* from the domain

Example 6:
Let *P(x)* denote *"x=x-3"*. Domain: all real numbers. Is $\exists x P(x)$ true?

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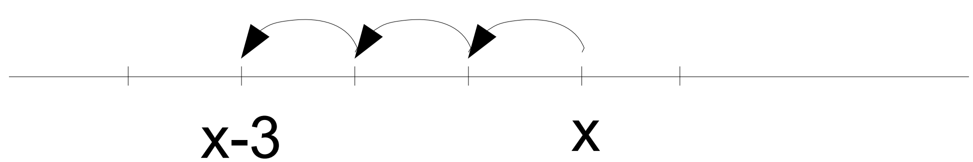
Example 6:
Let $P(x)$ denote *"x=x-3"*. Domain: all real numbers. Is $\exists x P(x)$ true?

Reasoning: It looks to be false. If we take a real number, and subtract a positive number from it, we'll get a smaller number.
It is probably more visible with a number line:



$x-3 < x$

1.6 Predicates and Quantifiers

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Example 7:
Let $P(x)$ stand for "$x > 10$". Domain: all real numbers. Is $\exists xP(x)$ true?

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Example 7:
Let $P(x)$ stand for "$x > 10$". Domain: all real numbers. Is $\exists x P(x)$ true?

Reasoning:
Following the definition, it will be enough for us to provide one value of x for which $x > 10$ is true.
For example, when $x=20$, $P(20)$: $20 > 10$ is true.
Therefore $\exists x P(x)$ is true

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Let $P(x)$ be "$x^2 \leq 16$". Domain consists of positive integers between 4 and 7, including. Is $\exists x P(x)$ true?

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Example 8:

Let *P(x)* be “$x^2 \leq 16$”. Domain consists of positive integers between 4 and 7, including. Is $\exists x P(x)$ true?

<u>Reasoning</u>:

What is the domain? 4, 5, 6, 7

What do we need to check? $P(4) \vee P(5) \vee P(6) \vee P(7)$

$4^2 \leq 16$	$5^2 \leq 16$	$6^2 \leq 16$	$7^2 \leq 16$	
T	F	F	F	but one T is enough

Therefore, $\exists x P(x)$ is true

1.6 Predicates and Quantifiers

One more quantifier: $\exists!$ -uniqueness quantifier

$\exists! x P(x)$ is "there exists a unique *x*, such that *P(x)* holds (or is true)"

denotation: $\exists!$ *or* $\exists_1$

$\exists! x P(x)$ is true, if *P(x)* is true for only one *x*

Example 9:
What is the truth value of the statement $\exists! x P(x) \rightarrow \exists x P(x)$?

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Example 9:
What is the truth value of the statement $\exists! xP(x) \rightarrow \exists xP(x)$?

Reasoning:
Let's read it: If there exists a unique *x* such that *P(x)* holds, then there exists at least one *x*, such that *P(x)* holds. - sounds good.

Let's do a truth table:

$\exists! xP(x)$	$\exists xP(x)$	$\exists! xP(x) \rightarrow \exists xP(x)$
T	T	T
F	F	T
F	T	T

Therefore, the statement $\exists! xP(x) \rightarrow \exists xP(x)$ is true.

Quantifiers with restricted domain

Let's assume that our domain is all real numbers.

examples:

$\forall x<0\ P(x)$ "for all x less than *0*, *P(x)* holds"

$\exists! x>0\ Q(x)$ "the exists a unique *x* grater than *0*, such that *Q(x)* holds"

$\forall x<0\ (x^2>0)$ "for all x less than *0*, x^2 is greater than *0*"

$\exists z>0\ (z^2=2)$ "there exists *z* grater than *0*, such that z^2 is equal to *2*"

By the way, the last two statements

$\forall x<0\ (x^2>0)$ "for all x less than 0, x^2 is greater than 0"

can be re-written as implication:

$\forall x<0\ (x^2>0) \equiv \forall x\ (x<0 \rightarrow x^2>0)$

"for all x, if x is less than 0, then x^2 is greater than 0"

$\exists z>0\ (z^2=2)$ "there exists z grater than 0, such that z^2 is equal to 2"

can be re-written as conjunction:

$\exists z>0\ (z^2=2) \equiv \exists z\ (z>0 \wedge z^2=2)$

"there exists z, such that z is grater than 0, such that z^2 is equal to 2"

1.6 Predicates and Quantifiers

Predicate logic – predicates + quantifiers

Precedence of quantifiers and logical connectives:

$\forall$, $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$

$\exists$

$\exists!$

Binding variables

$\exists x\ (2x-y=1)$

occurrence of the variable *x* is bound in the above statement,
occurrence of the variable *y* is not bound in the above statement.

Can you explain why?

Binding variables

$\exists x\ (2x-y=1)$

occurrence of the variable *x* is <u>bound</u> in the above statement,
occurrence of the variable *y* is <u>not bound</u> in the above statement.

Can you explain why?

- when a quantifier is used on a variable, say x, we say that this occurrence of the variable, x, is bound.

- an occurrence of a variable that is not bound by a quantifier, or set equal to a particular values, is free.

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- an occurrence of a variable that is not bound by a quantifier, or set equal to a particular values, is free.

! All the variables that occur in propositional function must be bound or set equal to a particular value to *<u>turn into a proposition</u>*.

A part of a logical expression to which a quantifier is applied is called *the scope of this quantifier*.

Binding variables

Example 10: show the scope of quantifiers; show bounded and free occurrences of the variables *x* and *y*.

a) $\exists x(2x-y=1)$

b) $\forall x\ (\ P(x) \vee Q(x)\ ) \wedge \exists x R(x)$

c) $\forall x\ (\ P(x) \vee \exists y\ Q(x,y)\ )$

Binding variables

Example 10: show the scope of quantifiers; show bounded and free occurrences of the variables *x* and *y*.

a) $\exists x \underbrace{(2x-y=1)}_{\text{the scope of } \exists}$

x is bound by ∃, *y* is free

b) $\forall x \underbrace{(\, P(x) \vee Q(x) \,)}_{\text{the scope of } \forall} \wedge \exists x \underbrace{R(x)}_{\text{the scope of } \exists}$

x is bound by; ∀ *x* is bound by ∃ *scopes do not overlap*

c) $\forall x \underbrace{(\, P(x) \vee \exists y \underbrace{Q(x,y)} \,)}$

x is bound by ∀, y is bound by ∃ *scopes overlap*

Logical Equivalences Involving Quantifiers

Statements involving predicates and quantifiers are *logically equivalent* if and only if (*iff*) they have the same truth values no matter which predicates are substituted into these statements and which domain is used for the variables in these propositional functions.

$S \equiv T$ (same notation as before, with propositions only)

Negating Quantified Expressions - De Morgan's Laws

$\neg\exists x P(x) \equiv \forall x \neg P(x)$

<u>Reading</u>: Let P(x): "x has taken a Calculus class", and domain: all students from our class,then

"It is not the case that there exists a student x that has taken a Calculus class"

is similar to saying

"there is no student in our class that has taken a Calculus class"

$\neg\forall x P(x) \equiv \exists x \neg P(x)$

<u>Reading</u>: Let P(x) be the same, then

"It is not the case that all students from our class has taken a Calculus class"

is similar to saying

"there is a student in our class that hasn't taken a Calculus class"

Translating from English into Logical Expressions

Example 13:
Let P(x) be the statement "x took a discrete math course", and Q(x) be the statement "x knows the computer language Python". Express each of these sentences in terms of P(x), Q(x), quantifiers and logical connectives. Let the domain for quantifiers consist of all students from Mathematics, CS, and Engineering majors.

a) There is a student who took a discrete math course.

b) There is a student who took a discrete math course, but doesn't know Python.

c) Every student either took a discrete math course or knows the computer language Python.

d) No student didn't take a discrete math course, but knows Python.

Translating from English into Logical Expressions

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$\exists x P(x)$

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a) There is a student who took a discrete math course.

$\exists x P(x)$

b) There is a student who took a discrete math course, but doesn't know Python.

$\exists x\, (P(x) \wedge \neg Q(x))$

c) Every student either took a discrete math course or knows the computer language Python.

d) No student took a discrete math course, but knows Python.

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$$\exists x P(x)$$

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$$\exists x\, (P(x) \wedge \neg Q(x))$$

c) Every student either took a discrete math course or knows the computer language Python.

$$\forall x\, (P(x) \vee Q(x))$$

d) No student took a discrete math course, but knows Python.

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d) No student took a discrete math course, but knows Python.

$$\neg\exists x(P(x) \wedge Q(x))$$

Example 14:
Suppose that the domain of the propositional function P(x) consists of the integers -2, -1, 0, 1. Express the given statements without using quantifiers (use only disjunctions, conjunctions, and negations):

a) $\exists x P(x)$

b) $\forall x \neg P(x)$

c) $\neg \forall x P(x)$

d) $\forall x\ ((x \neq -2) \rightarrow P(x)) \vee \exists x \neg P(x)$

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$P(-2) \lor P(-1) \lor P(0) \lor P(1)$

b) $\forall x \neg P(x)$

c) $\neg \forall x P(x)$

d) $\forall x\ ((x \neq -2) \to P(x)) \lor \exists x \neg P(x)$

Example 14:

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a) $\exists x P(x)$

$P(-2) \lor P(-1) \lor P(0) \lor P(1)$

b) $\forall x \neg P(x)$

$\neg P(-2) \land \neg P(-1) \land \neg P(0) \land \neg P(1)$

c) $\neg \forall x P(x)$

d) $\forall x \, ((x \neq -2) \rightarrow P(x)) \lor \exists x \neg P(x)$

Example 14:
Suppose that the domain of the propositional function P(x) consists of the integers -2, -1, 0, 1. Express the given statements without using quantifiers (use only disjunctions, conjunctions, and negations):

a) $\exists x P(x)$

$P(-2) \lor P(-1) \lor P(0) \lor P(1)$

b) $\forall x \neg P(x)$

$\neg P(-2) \land \neg P(-1) \land \neg P(0) \land \neg P(1)$

c) $\neg \forall x P(x)$

$\neg (P(-2) \land P(-1) \land P(0) \land P(1)) \equiv$ by De Morgan's Law

$\equiv \neg P(-2) \lor \neg P(-1) \lor \neg P(0) \lor \neg P(1)$

d) $\forall x\, ((x \neq -2) \rightarrow P(x)) \lor \exists x \neg P(x)$

Example 14:
Suppose that the domain of the propositional function P(x) consists of the integers -2, -1, 0, 1. Express the given statements without using quantifiers (use only disjunctions, conjunctions, and negations):

a) $\exists x P(x)$

$P(-2) \vee P(-1) \vee P(0) \vee P(1)$

b) $\forall x \neg P(x)$

$\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1)$

c) $\neg \forall x P(x)$

$\neg (P(-2) \wedge P(-1) \wedge P(0) \wedge P(1)) \equiv$ by De Morgan's Law

$\equiv \neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1)$

d) $\forall x ((x \neq -2) \rightarrow P(x)) \vee \exists x \neg P(x)$

$(P(-1) \wedge P(0) \wedge P(1)) \vee (\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1))$

Example 15:
For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

a) Everyone is studying discrete math

b) Every two people have the same father

c) No two different people have the same grandmother

Example 15:
For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

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Example 15:
For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

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domain_1: students in our class domain_2:kids from a daycare

b)Every two people have the same father

domain_1:Paul Smith, Rebeca Smith domain_2: students in our class

c)No two different people have the same grandmother

domain_1: Tom Cruise, Jim Carrey
domain_2: grandchildren of Benjamin Franklin