Sections 1.4-1.5 Practice Problems

1. Proof by truth tables that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

2. Show that $(\neg p \wedge (p \vee q)) \rightarrow q$ is a tautology without using truth tables.

3. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

Practice Problems

1. Proof by truth tables that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
T	T	F	F
T	F	F	F
F	T	F	F
F	F	T	T

Practice Problems

2. Show that $(\neg p \wedge (p \vee q)) \rightarrow q$ is a tautology without using truth tables.

$$
\begin{aligned}
(\neg p \wedge (p \vee q)) \rightarrow q &\equiv ((\neg p \wedge p) \vee (\neg p \wedge q)) \rightarrow q && \textit{by De Morgan laws (1)} \\
&\equiv (\mathbf{F} \vee (\neg p \wedge q)) \rightarrow q && \textit{by Negation laws (8)} \\
&\equiv (\neg p \wedge q) \rightarrow q && \textit{by Identity laws (7)} \\
&\equiv \neg(\neg p \wedge q) \vee q && \textit{by (11)} \\
&\equiv \neg\neg p \vee \neg q \vee q && \textit{by De Morgan laws (1)} \\
&\equiv p \vee \neg q \vee q && \textit{by Double negation law (2)} \\
&\equiv p \vee \mathbf{T} && \textit{by Negation laws (8)} \\
&\equiv \mathbf{T} && \textit{by Domination laws (6)}
\end{aligned}
$$

Practice Problems

3. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

$(p \vee q) \rightarrow r \equiv \neg(p \vee q) \vee r$ *by (11)*

$\equiv (\neg p \wedge \neg q) \vee r$ *by De Morgan laws (1)*

$\equiv (\neg p \vee r) \wedge (\neg q \vee r)$ *by Distributive laws (10)*

$\equiv (p \rightarrow r) \wedge (q \rightarrow r)$ *by (11)*