Practice Problems

Example 1.

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if *H(x)*: "*x is honest*" and the domain of *x* is all politicians.

b) Negate $\forall x(x^2 > x)$

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$\neg\exists x H(x) \equiv \forall x \neg H(x)$

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Negated *"There is an honest politician"* : $\neg\exists x H(x)$
$\neg\exists x H(x) \equiv \forall x \neg H(x)$: *"All politicians aren't honest"*

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b) Negate $\forall x(x^2>x)$:

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$\neg\forall x(x^2 > x) \equiv \exists x\neg(x^2 > x) \equiv \exists x(x^2 \le x)$

Puzzle by Smullyan.
There is an island that has two kinds of inhabitants. Knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people A and B. What are A and B, if A says "I am a knave or B is a knight" and B says nothing.

Solution:
A: "I am a knave or B is a knight"
B: nothing

Let's introduce two predicate variables
p: "A is a knight"
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Now, let's assume that A is a knight. He should be telling the truth. Therefore, we have:

$$p \wedge (\neg p \vee q)$$

stating that A is a knight (our assumption)

this is what A says (since he is supposed to tell truth, this compound proposition should be true)

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stating that A is a knight (our assumption) → p

this is what A says (since he is supposed to tell truth, this compound proposition should be true) → $(\neg p \vee q)$

Note, that although $\neg p$ is False, it doesn't mean that A is lying.

We need to look at the ENTIRE phrase: $\neg p \vee q$ -whether the entire phrase is True or False. If it is True – he is telling truth.

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Now, let's assume that A is a knight. He should be telling the truth. Therefore, we have:

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So, it turns out that p is True, and q is True, i.e. A is a knight, and B is a knight