

Practice Problems

Example 1.

a) Negate “*There is an honest politician*” using Predicate Logic, if $H(x)$: “*x is honest*” and the domain of x is all politicians.

b) Negate $\forall x(x^2 > x)$

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Negated “*There is an honest politician*” : $\neg\exists xH(x)$

$\neg\exists xH(x) \equiv \forall x\neg H(x)$

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“*There is an honest politician*” : $\exists xH(x)$

Negated “*There is an honest politician*” : $\neg\exists xH(x)$

$\neg\exists xH(x) \equiv \forall x\neg H(x)$: “*All politicians aren't honest*”

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b) Negate $\forall x(x^2 > x)$:

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b) Negate $\forall x(x^2 > x)$:

$$\neg \forall x(x^2 > x) \equiv \exists x \neg(x^2 > x) \equiv \exists x(x^2 \leq x)$$

Puzzle by Smullyan.

There is an island that has two kinds of inhabitants. Knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people A and B. What are A and B, if A says “I am a knave or B is a knight” and B says nothing.

Solution:

A: “I am a knave or B is a knight”

B: nothing

Let's introduce two predicate variables

p: “A is a knight”

q: “B is a knight”

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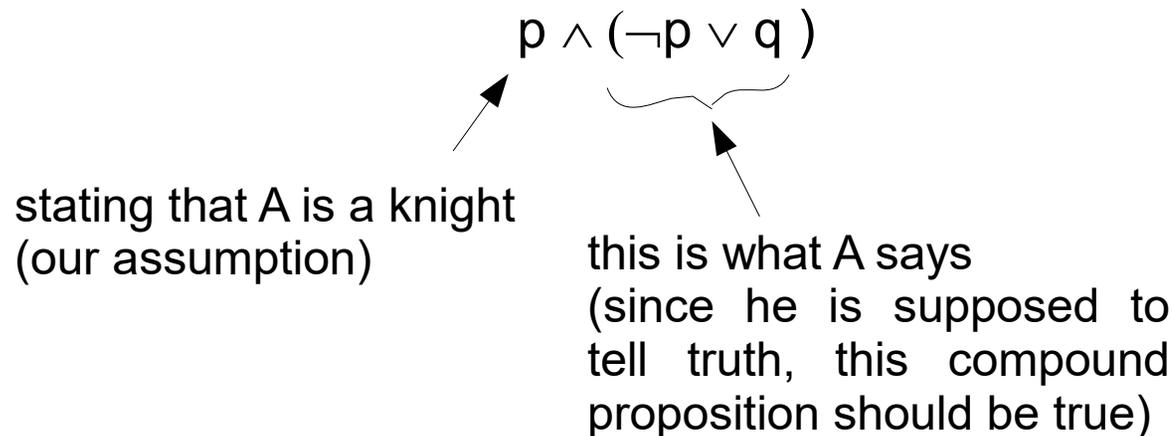
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Now, let's assume that A is a knight. He should be telling the truth. Therefore, we have:



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Now, let's assume that A is a knight. He should be telling the truth. Therefore, we have:

$$p \wedge (\neg p \vee q)$$

stating that A is a knight
(our assumption)

this is what A says
(since he is supposed to
tell truth, this compound
proposition should be true)

Let's see: $p \wedge (\neg p \vee q)$

T F

q must be True, in order for the entire
compound proposition to be true

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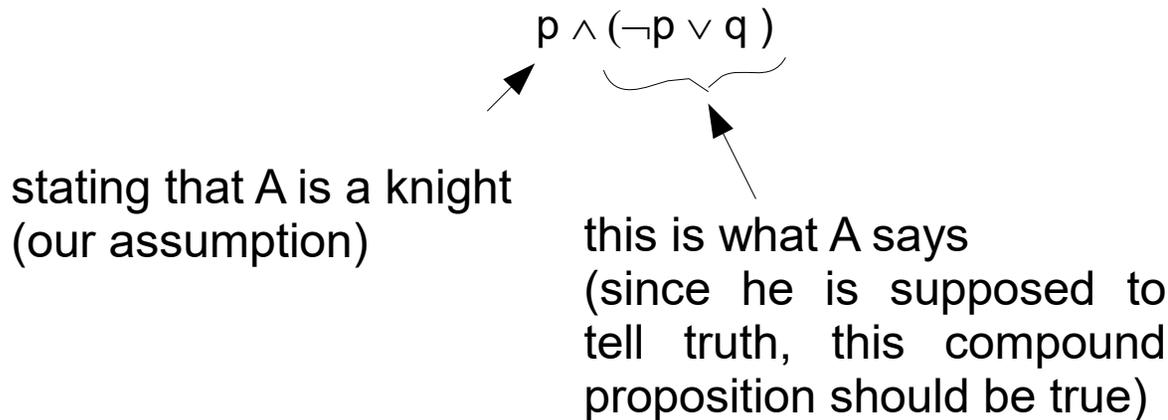
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Now, let's assume that A is a knight. He should be telling the truth. Therefore, we have:



Note, that although $\neg p$ is False, it doesn't mean that A is lying.

We need to look at the ENTIRE phrase: $\neg p \vee q$ -whether the entire phrase is True or False. If it is True – he is telling truth.

Let's see: $p \wedge (\neg p \vee q)$
 $\quad \quad T \quad F$
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Now, let's assume that A is a knight. He should be telling the truth. Therefore, we have:

$$p \wedge (\neg p \vee q)$$

So, it turns out that p is True, and q is True, i.e. A is a knight, and B is a knight